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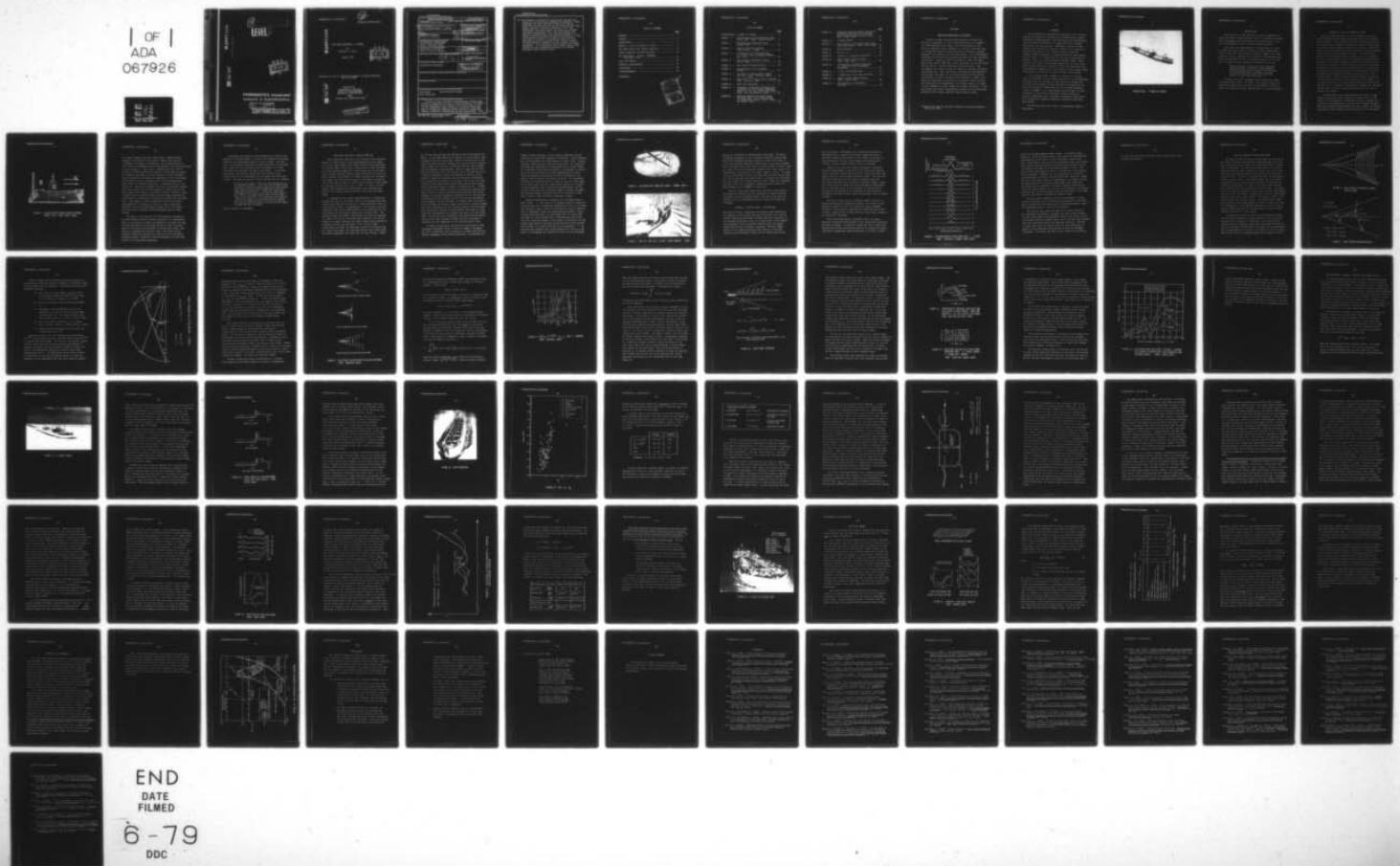
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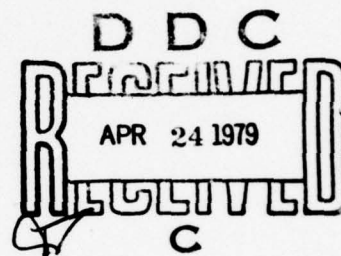
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SHIP WAVE RESISTANCE - A SURVEY

By
Marshall P. Tulin

August 1978



Presented at the U.S. National Congress on Applied Mechanics,
UCLA, July 1978

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ABSTRACT

Ship Wave Resistance - A Survey*

Studies of ship resistance have generated concepts and methods of importance for all of fluid mechanics; for example: the understanding of the origin of fluid resistance in its various components; model testing as a science; similitude laws in testing and analysis; group velocity in dispersive systems; asymptotic integration; and thin body theory. Here, developments including the earliest are surveyed and placed in some historical perspective: Newton (b. 1642) to Froude (b. 1810); The Short Time of Wm. Froude (1867-79); Kelvin, Havelock, and the Far Field; The Near Field: Michell, Havelock, Guilloton, et al. The development and current status of theory are surveyed critically, concluding with: Slow Ship Theory, and Numerical Hydrodynamics. The growing convergence of recent theoretical predictions and observations is noted, and particular attention is drawn to the essential importance in practice of non-linear effects on both wave generation and propagation; the relative neglect in theory of wave breaking is noted, despite its common occurrence. It is speculated that useful computational methods for wave resistance prediction will become available within another decade.

* Presented at the U.S. National Congress on Applied Mechanics, UCLA, July 1978.

FOREWORD

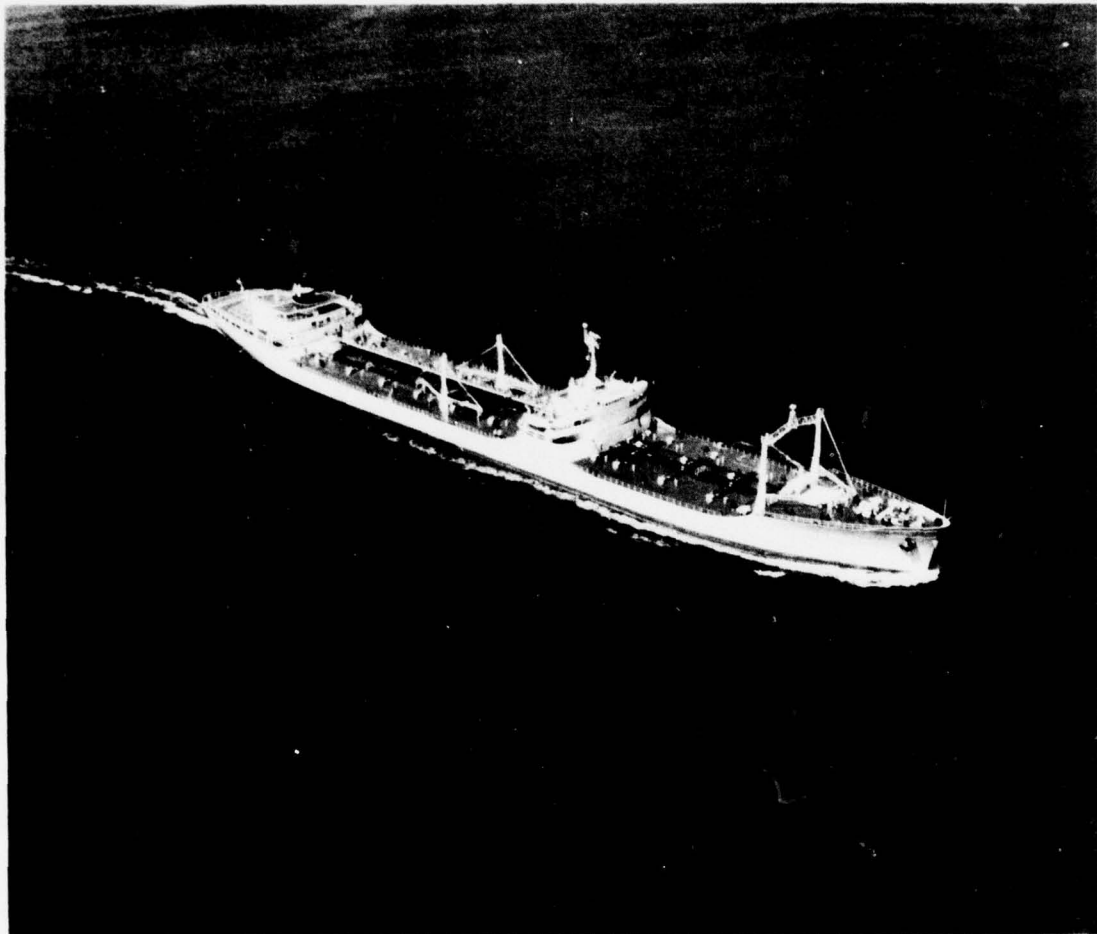
The prediction and thorough understanding of the relation between the shape of a ship and its resistance poses a great challenge for hydrodynamics, both theoretical and experimental, and the published literature is large; about 500 citations appeared in the last 15 years. Naturally a subject of this interest has earlier been surveyed, either in its entirety or in particular aspects. The recent excellent review by Wehausen (1973) in *Advances in Applied Mechanics* comes immediately to mind. It is expository in style and encyclopedic in breadth. There is a Soviet book by Kostyukov (1959). More recently, the subject was reviewed again, but in a more piecemeal fashion, by various authors in their contributions to an "International Seminar on Wave Resistance," held in Tokyo (1976), subsequently reviewed by the present author, Tulin (1976).

In the present work I attempt to evaluate in critical terms the situation of the subject, and also to put developments, and especially the earliest, in some historical perspective, particularly as studies of ship resistance have generated results and concepts of importance for all of fluid mechanics. I give as examples: the understanding of the origin of fluid resistance in its various components; model testing as a science; similitude laws in testing and analysis; group velocity in dispersive systems; asymptotic integration; and thin body theory.

For the most part I will refer to displacement ships in deep water.

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FRONTISPIECE: A TANKER ON COURSE

INTRODUCTION

Ships have for a long time been of central importance in trade and warfare between nations, and it is therefore no wonder that encouragement was early given to the application of scientific knowledge to their construction and operation.

That great court mathematician, Leonhard Euler (b. 1707-d. 1783), published several treatises on naval architecture (1749, 1773), the latter entitled "Théorie Complete de la Construction et de la Manoeuvre des Vaisseaux," subsequently translated into English in 1776, an uncommon practice at the time. On the subject of ship resistance, Euler had this to say:

"From good Models in Miniature which represent Vessels exactly as they are, very important Experiments upon the Resistance of Vessels may be very usefully made; and which is so much the more necessary, as the Theory upon the Subject is still very defective."

It happens that this statement is precisely true today, two hundred years later. Of course, the defective theory of which Euler spoke is not the same as the defective theory of today, ours being much more sophisticated — as I shall explain.

NEWTON (b. 1642) TO FROUDE (b. 1810)

Euler no doubt alluded to Sir Isaac Newton's theory of fluid resistance based on the notion that resistance resulted from the impact on the forebody of fluid particles directly in the body's path, leading to a differential equation from which the solid of least resistance could be calculated. Newton was specific in his Principia that this forebody of least resistance 'may be of use in the building of ships.' Of course, Euler understood the defective nature of this idea, for d'Alembert in 1752 and Euler himself in 1755 had published demonstrations of what we call today d'Alembert's paradox concerning the zero resistance of bodies in uniform motion, and Euler consequently specifically pointed out to naval architects of the time that the whole of the underwater portion of the hull had to be considered when dealing with resistance problems, and not the forebody alone. Euler's remarks notwithstanding, the history of naval architecture has led one observer, Stoot (1959), to the conclusion that "the shipbuilding industry believed in the efficacy of this (Newton's) concept right through the eighteenth century, and it was not finally dismissed until the time of Froude (circa 1870)."

In the 200-odd years between Newton and Froude, a large number of individual model tests were carried out by a wide variety of investigators, invariably utilizing a towing cable driven by a falling weight, as in the case of Samuel Fortrey-1650, and Benjamin Franklin-1768, see Rouse (1976), who studied the effect of water depth on the resistance of a 6 inch model

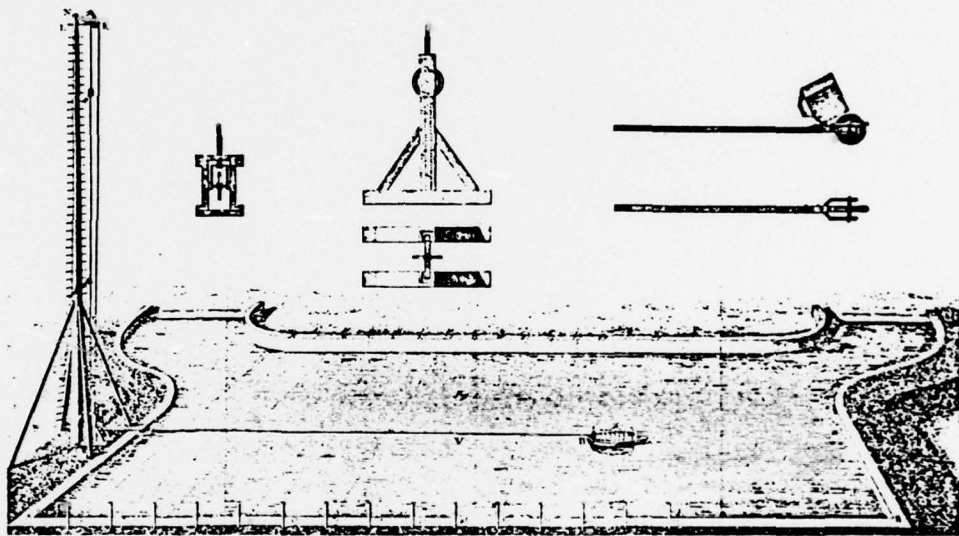


FIGURE 1 - FALLING WEIGHT TOWING SYSTEM IN POND,
PARIS, 1775. FROM: STOOT (1959)

in a small trough of his own construction. Among the most elaborate and careful of the earlier studies were those of d'Alembert, the Marquis de Condorcet, and the Abbé Bossut in 1775, using an ornamental pond on the grounds of the Ecole Militaire in Paris, Figure 1. Their results demonstrated that the resistance of ships increased not exactly as the square of speed, as thought at the time, but more quickly, and they confirmed Euler's prediction concerning the importance of the stern shape and Franklin's earlier demonstrations that decreasing water depth seemed to increase resistance — a finding we know is too simplistic. Much more extensive and better instrumented studies were later carried out by Mark Beaufoy in a large dock of 400 foot length, near London in the period 1791-98. Beaufoy's tests included those on friction planks and were highly important in quantifying the significance of frictional effects for the first time. Then during the period 1834-1840, John Scott Russell investigated the resistance of ships in canals, using both horsepower and the falling weight system, and in the process discovered what he termed "the great primary wave of translation."

The same J. Scott Russell, in 1870 chairing a meeting of the (now) Royal Institute of Naval Architects, an organization which he had helped to found, and himself the inventor and very forceful proponent of an erroneous theory of ship resistance based on his earlier observations of shallow water waves, confessed to designing and testing a ship's hull based on the solution of Newton's equation for solids of least resistance. Thereupon ensued a discussion from the audience as to the difficulty of solving these equations.

Ironically, the purpose of the meeting was to discuss the proposal of William Froude, a retired civil engineer and already known for his successful analysis of ship rolling. This proposal to the British Admiralty, already approved in the previous year, asked for the construction and development of a model basin for the scientific testing of ship models. It was most vigorously, even testily, opposed by Scott Russell, as he claimed that his own experiments did not agree with full scale results. The quiet and earnest Froude replied from the audience:

"I did not come here to make any long explanation to the meeting to-night. I see that the feeling of the meeting is very much against experiments with models, but I must say that my own experience leads me to judge quite differently. I think the reason why experiments with models have hitherto been found to be a failure, and have misled those who have made them, as to the effect to be expected with regard to a full-sized ship, is, that attention has not been paid to the relation which should subsist between the speed at which the model is moved, and the speed at which the ship is moved."

This was Froude's thunderbolt.

THE SHORT TIME OF WM. FROUDE (1867-79)

Isaac Newton had observed in his Principia that the phase speed of a surface wave varies approximately with the square root of the wave length, and in 1852 Professor Ferdinand Reech (b. 1805-d. 1880) of the Ecole d'application du Génie Maritime in Paris published amongst his lecture notes the law of similitude for gravitational effects known justifiably in France as Reech scaling, and elsewhere as Froude scaling, the law expressing simply the fact that the speed of the model be scaled to that of the ship so that the ratio of wave length to hull length be identical in each case, i.e., $U \sim \sqrt{L}$. Of course, the law follows from dimensional considerations alone ignoring the effects of viscosity, surface tension, cavitation and finite depth.

It remained for Wm. Froude experimentally to confirm and utilize this law of similitude, which he had independently discovered about 1867 in the process of conducting model tests in a pond near his home. He enunciated it very clearly, with corroborative data, in his 1868 proposal to the Admiralty for the construction of an enclosed towing tank and model shop. The proposed budget was 2000£, which Froude justified by pointing out "Were it to result in beneficially taking 10 feet off the length of a single ironclad, the whole outlay would probably be recouped at once," see the Papers of Wm. Froude (1955). A year later, in 1869, the Admiralty accepted the proposal but on a fixed cost basis, and requiring further that Froude throw in without additional cost some tests on ship rolling. Froude

was at this time 59 years old and was to live only another ten years. He was much aided in his efforts by his son Robert, who continued them after his death. Froude's contributions to the subject of ship resistance during these ten years have constructively shaped the course of all future work and understanding and can hardly be overestimated. And it is all subjects benefiting from model tests and from the use of scaling laws, particularly Aeronautics, which are indebted to Froude, for he without a doubt and virtually with his own hands established the testing of scaled models as a science, and at the same time introduced into engineering consciousness the practical importance and proper usage of scaling or similitude laws. Trained in mathematics at Oxford, he was a fine practical engineer and could conceive, design, and handicraft by himself the various refined and accurate equipments necessary for the construction of models, for their towing, and for the accurate measurement and recording of forces on them. He brought model testing indoors, replaced the falling weight system with a towing carriage and rails, introduced the use of models built of hard paraffin wax, and devised suitable cutting machines for them; his original model dynamometer and recorder were used in Britain until 1950. He also devised a dynamometer and instituted procedures for the full scale testing of ships, which he proceeded to carry out for the comparison of the data with model tests.

In order to succeed in the successful prediction of ship resistance from model tests, it was not enough to recognize the law of similitude, and to construct a towing tank with its diverse equipment, it was further necessary to understand the

origin of ship resistance in its various components and the necessity to scale these components separately. Froude proposed that the resistance of a ship consists of three items: surface friction, eddy-resistance, and wave resistance. He understood perfectly the importance of streamlining and of "easy" shapes. His conception of the eddy-resistance, as incidental to surface friction, and resulting in a slight unbalancing of perfect fluid streamlines and pressures exactly corresponds to our present view of what we call form resistance. The frictional resistance of the ship hull he related to that of a fine plank of the same length and area, empirically adjusted for effects of ship roughness, and he understood experimentally that the friction decreases with increasing length of surface (remember that Reynolds number scaling had yet to be invented and was only related to skin friction by Rayleigh in 1900).

Froude described wave resistance in a way impossible to fault today" "... the ship in its passage along the surface of the water has to be continually supplying the waste of an attendant system of waves, which, from the nature of their constitution as independent waves are continually diffusing or transmitting themselves into the surrounding water or, where they form what is called broken water, crumbling away into froth. Now waves represent energy, or work done; and therefore all the energy represented by the waves wasted from the system attending the ship, is so much work done by the propeller... ." He described the familiar wave system produced by a ship, as composed of "diverging" and "transverse" waves, in the following

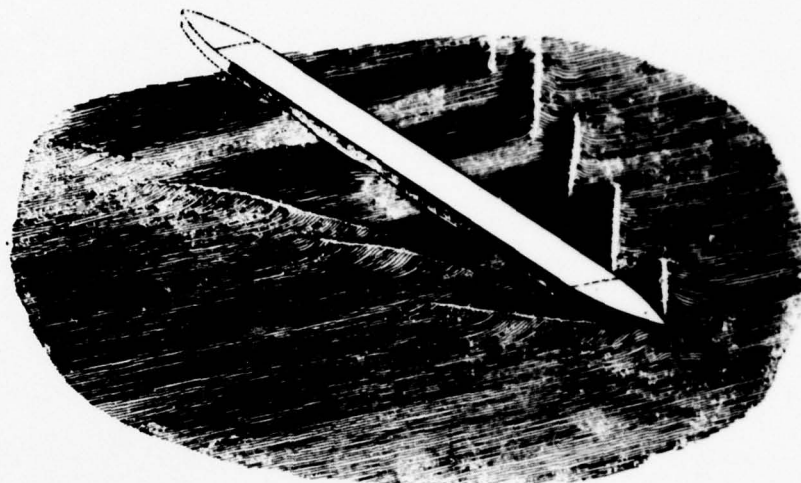


FIGURE 2 - DIVERGING AND TRANSVERSE WAVES. FROUDE (1877)

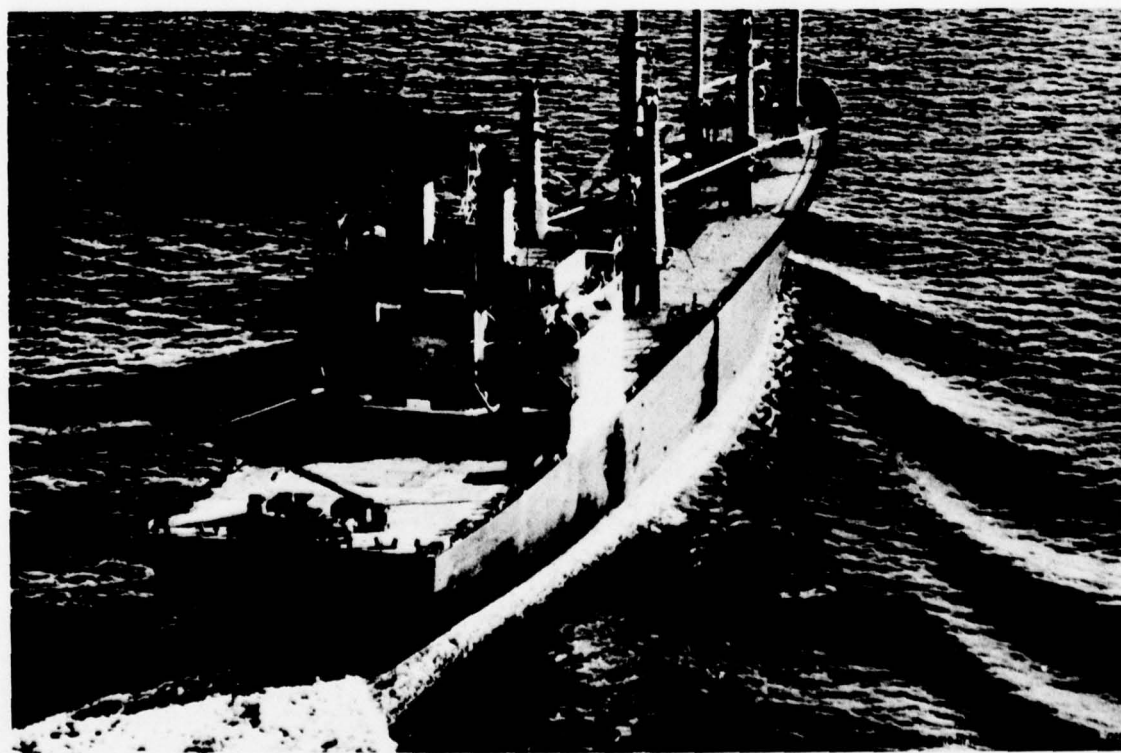


FIGURE 3 - ROLL-ON, ROLL-OFF, 25 KNOT, CARGO CARRIER. (1968)

way and so introduced the nomenclature used today, "the whole wavemaking resistance is the resistance expended in generating first the diverging bow waves, which as we have seen cease to act on the ship once they have rolled clear of the bow; secondly, these transverse waves, the crests of which remain in contact with the ship's side, and thirdly the terminal wave, which appears independently at the stern of the ship," Figures 2 and 3. He demonstrated conclusively in tests of ship's with parallel middle body the role of bow and stern interaction in creating the oscillatory wave resistance curve characteristic of most ships, and explained the peak in the wave resistance coefficient which occurs at a Froude number, $F_L = (U/\sqrt{g\ell})$, of about 0.5.

Froude is best known among naval architects today for his extrapolation procedure for the estimation of ship resistance from model tests which begins with a division of the total resistance into two components, which he called frictional and residuary:

$$R(\text{Total}) = R(\text{Frictional}) + R(\text{Residuary})$$

The first of these, comprising the turbulent skin friction to be estimated for both model and ship from plank tests and the second, comprising both eddy-resistance and wave resistance to be estimated from model tests at the appropriate Froude number. Many phenomena could conceivably interfere with the proper functioning of this procedure, including: Reynolds number effects on the eddy-resistance; shape effects on the skin friction; and interference effect of the viscous flow on the

wave resistance. Furthermore, the residuary resistance is determined in the absence of the ship's propeller, and is therefore absent of any effects of the latter on the wave or eddy resistance. Nor are scale effects on breaking given account, and here surface tension could be important.

Suffice it to say, that despite the neglect indicated, Froude's method works so well that it is still used today by all of the world's model basins of which I know, and these now number about 70, there being 30 alone in Japan. The only significant change has been the formal recognition that the frictional resistance is a function of Reynolds number, allowing a correlation of plank data utilizing modern formulations for the friction curve, à la von Karman, as carried out in 1934 by Karl Schoenherr of the Experimental Model Basin in Washington (the EMB had been constructed by Adm. David W. Taylor in 1900).

How much of the success of Froude's method is due to a mutual cancellation of neglected effects? The answer to this important question is not well understood today, although scattered results provide quantitative information about some of them - shape effects on skin friction, for example, found to be usually not important.

Model basin experimental techniques have, of course, advanced continually since Froude's time, especially in measurement and recording. In addition, within the last 30 years the possibility has been practically realized to dissect the ship's resistance through wave probe measurements (to which I return

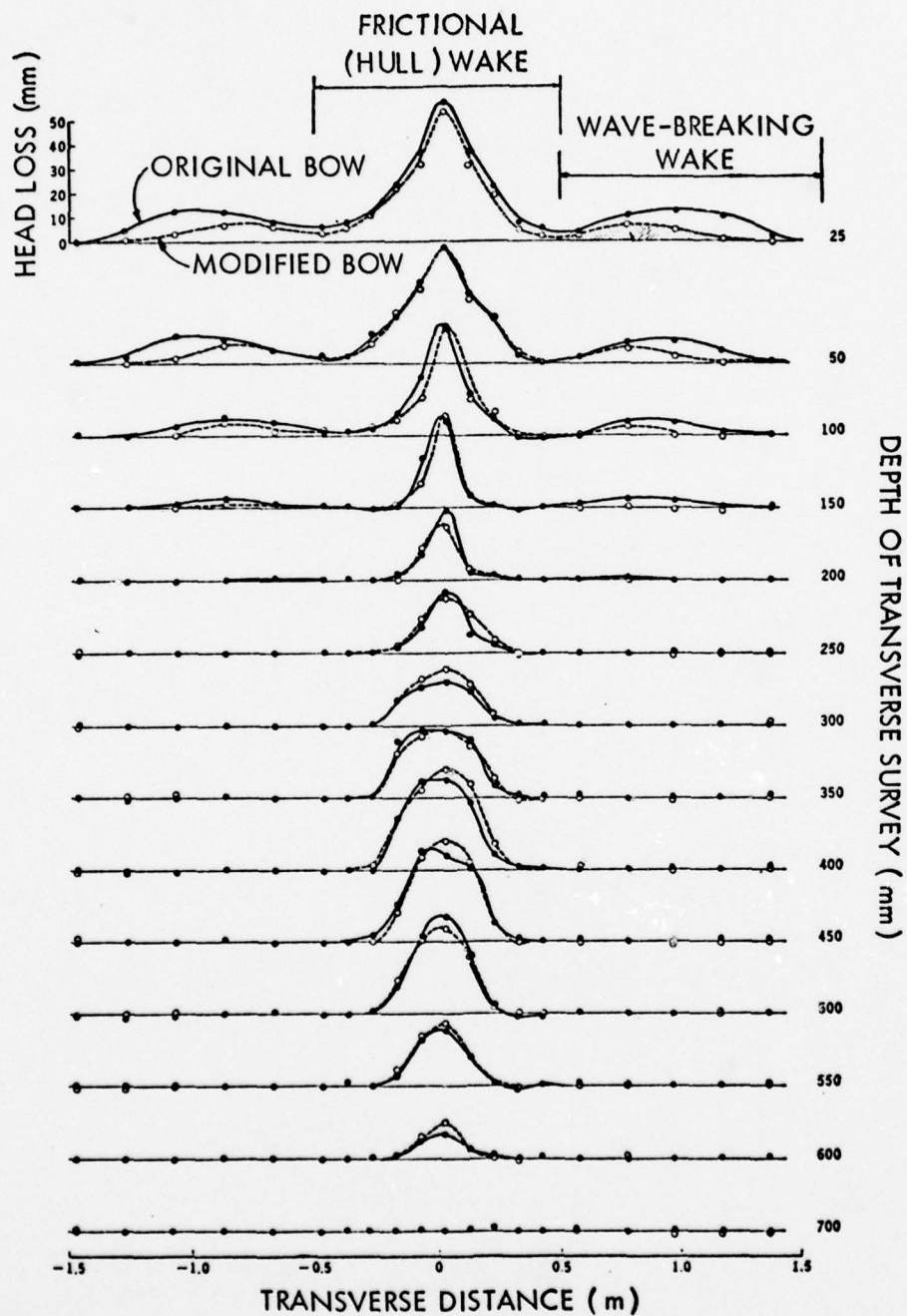


FIGURE 4 - DISTRIBUTION OF TOTAL HEAD LOSS, $F = 0.237$
FROM: TANIGUCHI, TAMURA, BABA (1971)

later) and through momentum wake surveys. A transverse wake survey at a short distance behind a ship allows the experimental determination of the total viscous resistance, friction and form, Tulin (1951), just as in the aerodynamics case, Betz (1925). The technique has been utilized, first by Jin Wu (1962) and his colleagues, and lately in an increasing number of cases. The method could conceivably be used as the basis for a new extrapolation procedure. However, its real importance is to dissect the dynamometer resistance and more definitely assign its origin. This use of the wake survey is exemplified in its highly successful application by the Japanese Baba (1969), who showed conclusively that the anomolous low speed residuary resistance of full ships (like tankers) was not due to wavemaking as previously believed, but to wave breaking around the bow region of the ship, Figure 4; his work was extended by Townsin (1972).

The wake survey technique, both the measurement of total viscous resistance and of the detailed flow pattern in the wake, will certainly find increasing use, for hull improvement, screw and appendage design, as well as for research.

The viscous flow around the ship is of vital concern to the appendage and propeller designer, who must produce designs to work in the highly non-homogeneous wake at the stern, to provide design thrust and rudder forces and at the same time minimize noise and vibrations, which can sometimes be severe. However, it seems clear that the central issue for ship resistance research is the estimation and reduction of wave and breaking resistance. The reason for this is that frictional resistance

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is much more predictable for the designer than are wave and breaking resistance.

KELVIN, HAVELOCK, AND THE FAR FIELD

The story of wave resistance theory begins in the time of Wm. Froude, who knew some important mathematicians of his day and seems to have had a marked and stimulating effect on their work on waterwaves. In 1873, three years prior to his first published work on group velocity, Wm. Stokes received a letter from Froude describing tank observations in precise and provocative terms. He wrote of a group of waves, how the group as a whole advances with a less velocity than that of the waves composing it, wave crests advancing through the group in its motion and appearing to die away at the front while new ones were formed at the rear (quoted by Havelock, 1934). And doubtless, his early observations of ship wave patterns, supplemented by those of his son Robert, stimulated the important theoretical account first given by Wm. Thomson, Lord Kelvin, (1887) of the waves produced by a pressure point. The Froudes and Thomson were friends; Froude acknowledges his debt to Thomson not only for explanations of hydrodynamic phenomena but also in the design of simple machines for the model basin, a task in which Thomson was said to be "most acute."

Kelvin's wave pattern as well as the phenomena of group velocity which underlies its construction, are of central importance for the subject of ship waves. Not only does the kinematic pattern reproduce many of the generally observed features of the wave patterns about real ships, but as generalized by Havelock (1934), it provides a description of the far field, which would seem to be asymptotically exact, Figures 5 and 6.

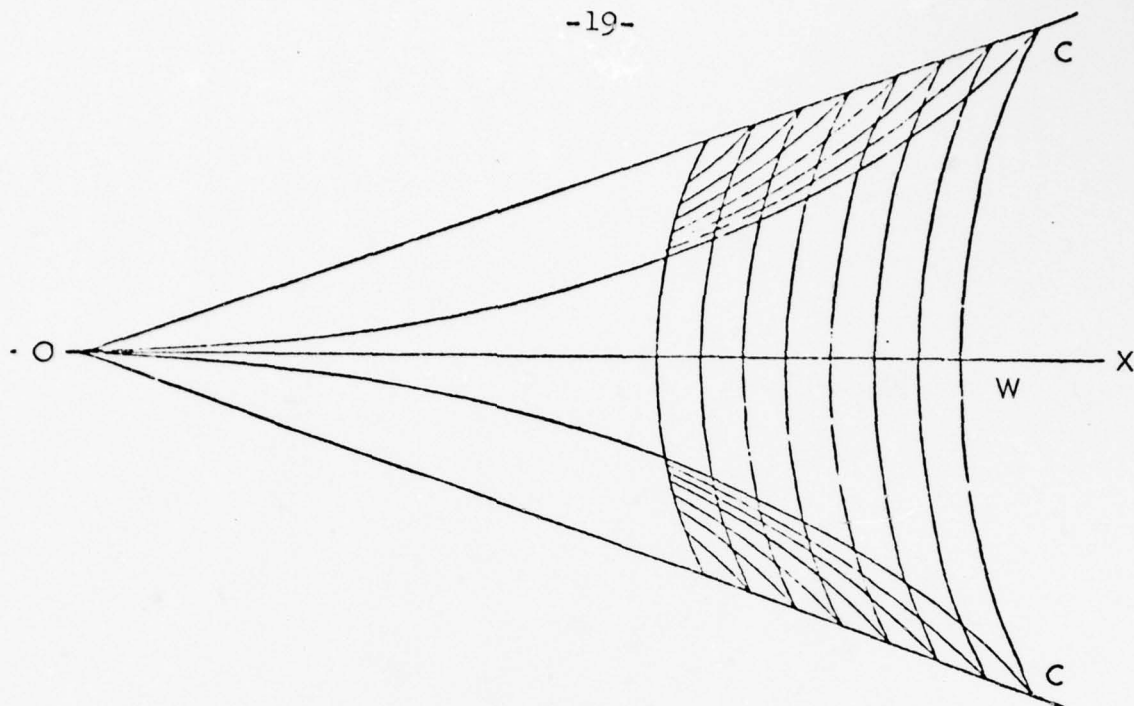


FIGURE 5 - WAVE PATTERN (ISOPHASAL CURVES),
KELVIN (1887)

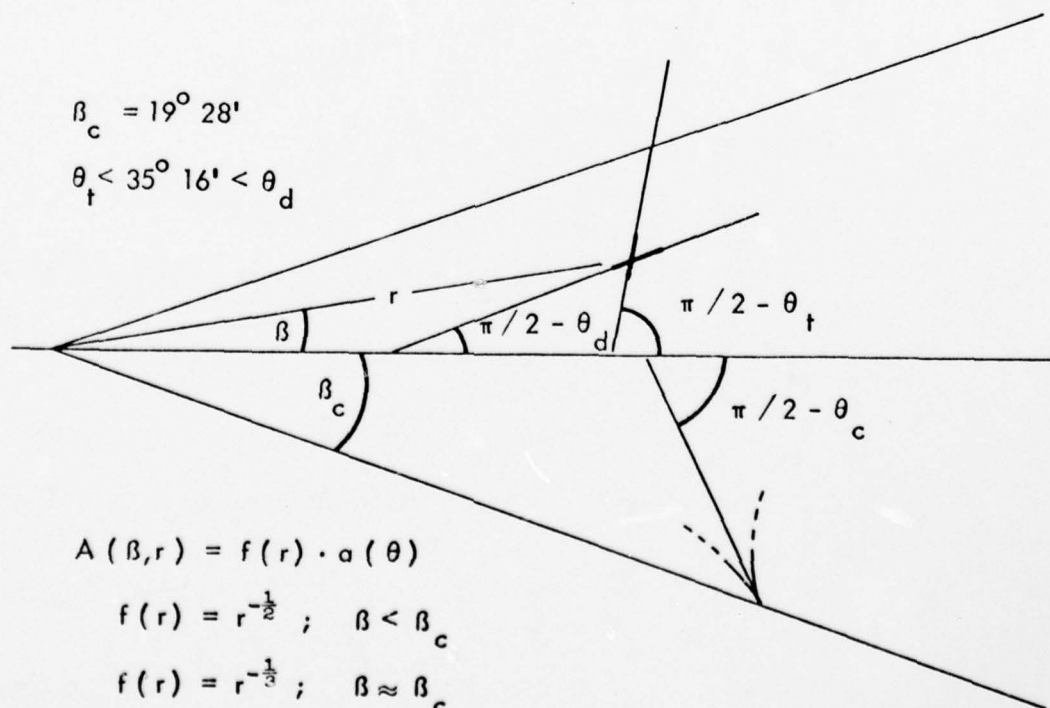
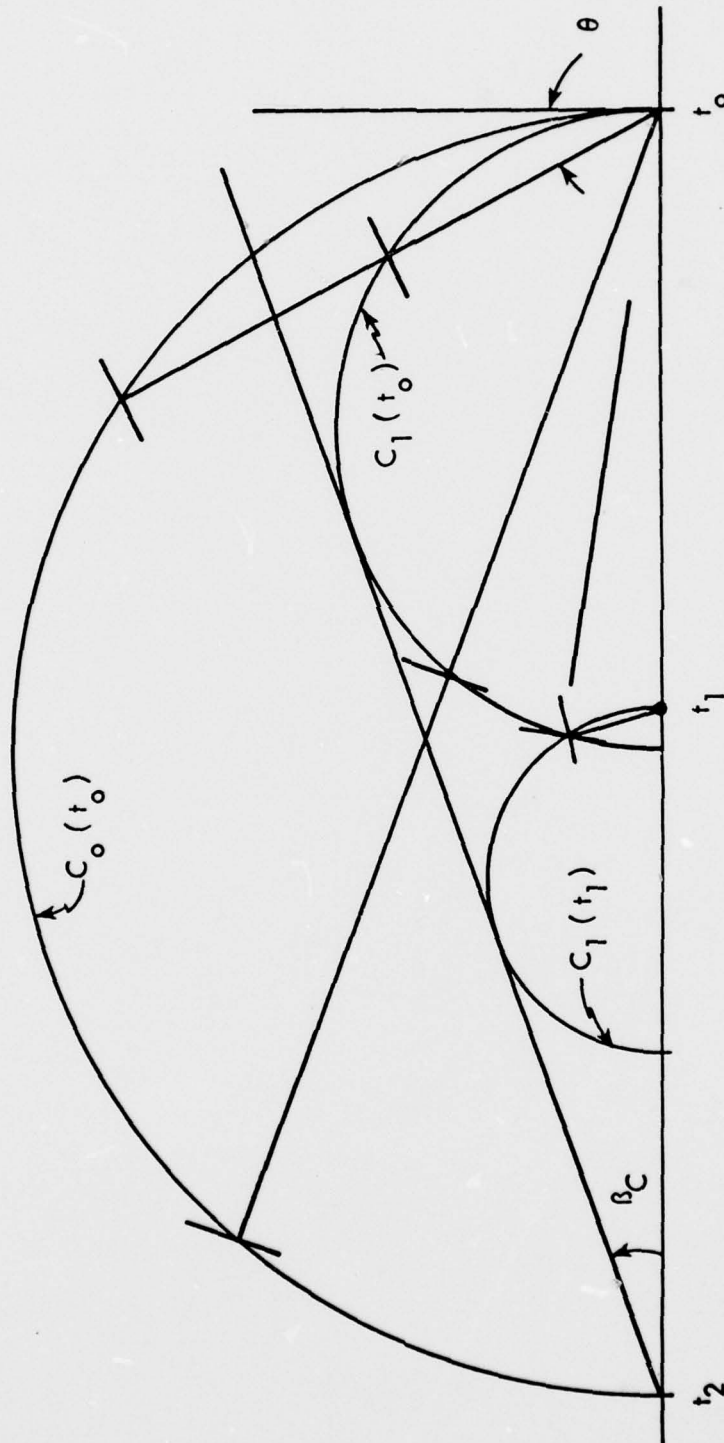


FIGURE 6 - WAVE PATTERN CHARACTERISTICS

Kelvin's pattern was calculated explicitly by him for a pressure point using his method of asymptotic integration, which he had invented for such purposes. The pattern possesses some remarkable features:

- (a) Two waves with crest angles θ_d and θ_t exist at each point within and only within an angle to the direction of motion, $\beta_c = 19^\circ 28'$.
- (b) These crest angles depend only upon the angle β .
- (c) Excepting a small neighborhood of β_c , the amplitude, A , of the wave crests at any angle β depend only on r , the radial distance from the pressure point; i.e., $A(\beta, r) = f(r) \cdot a(\theta)$.
- (d) The crest angles coincide at $\beta = \beta_c$, where $\theta(\beta_c) = 35^\circ 16'$, while $\theta_d > 35^\circ 16'$ and $\theta_t < 35^\circ 16'$.
- (e) Simple energy considerations lead to the conclusion that $f(r) = r^{-\frac{1}{2}}$.

Near the caustic at β_c in a boundary layer which grows as $r^{1/3}$, Ursell (1960a), the wave amplitude decays only as $r^{-1/3}$ as shown by Havelock (1908). All of these conclusions, excepting the caustic behavior, may be derived by application of Huyghen's principle and the principle of group velocity, specific to the appropriate dispersion relation (deep water, shallow water, etc.). The application of the first of these is tantamount, in the case of uniform speed, U , of the disturbance, to the requirement that each wave component appears



$$c_p = U \cos \theta$$

$$c_g = d\omega / d\kappa$$

$$c_g = \frac{1}{2} c_p \text{ (DEEP WATER)}$$

FIGURE 7 - CONSTRUCTION OF THE ISOPHASAL CURVES

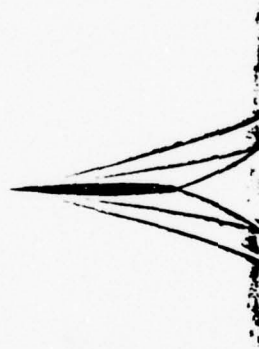
stationary when viewed in the frame of reference of the disturbance, i.e., $c_{\text{phase}} = U \cos \theta$. Thus the waves ($0 < \theta < \pi/2$) created at time t_0 would in a naive view appear later on the circle C_0 , whereas interference phenomena due to wave dispersion result in the wave fronts appearing on the circle C_1 , dependent on the magnitude $c_{\text{group}} = d\omega/dk$. In the case of deep water, $c_g = \frac{1}{2}c_p$, the case shown in Figure 7. This construction leads immediately to the calculation for β_c , the angle of the caustic, and $\theta(\beta_c)$. Further, it is easy to see that for any point on $\beta < \beta_c$, two and only two circles $C_1(t_0)$ and $C_1(t_1)$ intersect, representing the fronts for waves originating at two different times, and leading to the transverse waves $[C_1(t_0)]$ which were born longer in the past, and the younger divergent waves $[C_1(t_1)]$.

This simple method of calculation was not used early, as far as I know, but was given by Lighthill (1956) and has been applied independently in essentially the same way by Stoker (1957) to the case not only of uniform, but non-uniform motion. It may also be applied to show the profound effect of water depth on the kinematical wave pattern, a phenomenon first described experimentally by Marriner (1905), Figure 8, and theoretically by Havelock (1908), corrected by Inui (1934). Thomas Havelock's work, including a treatment of the caustic, was the very first of about 50 important papers on ship waves which he contributed over a period of 50 years, during which he came close to dominating the subject. His works are collected (1963).

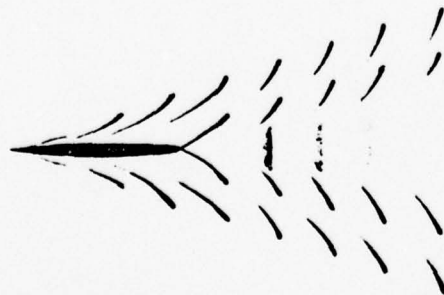
Kelvin's calculation refers specifically to a vertical force applied to the water surface at a point, a singularity



WAVE FORMATION ABOVE CRITICAL SPEED



WAVE FORMATION AT CRITICAL SPEED



WAVE FORMATION BELOW CRITICAL SPEED

FIGURE 8 - THE EFFECT OF WATER DEPTH ON WAVE PATTERNS
FROM: MARRINER (1905)

much later recognized by Havelock (1934) to correspond as well to a submerged horizontal doublet when brought to the water surface. In this case:

$$a(\theta) = (\text{const}) \sec^4 \theta$$

In the case of a horizontal dipole equivalent in unbounded flow to a sphere of radius r_s , submerged in water of depth f , and moving with speed U , Havelock (1934) showed that:

$$a(\theta) = 2 \kappa^2 r_s^3 \sec^4 \theta \cdot e^{-\kappa f \sec^2 \theta}$$

as shown in Figure 9. ($\kappa = g/U^2$, is the fundamental wave number). Notice the profound effect of submergence on the wave spectra, a fact of great importance for ship theory.

In the same paper, written almost 50 years after Kelvin's first work, Havelock pointed out that every far field wave pattern (here Havelock never referred to the far field, preferring to speak of "free" wave patterns) could be represented in the case of deep water by a continuous spectrum of planar waves, stationary in the Huyghen's sense, leading to an expression for the surface elevation, ζ :

$$\zeta = \int_{-\pi/2}^{+\pi/2} \left[S(\theta) \sin + C(\theta) \cos \right] (\kappa \sec^2(x \cos \theta + y \sin \theta)) d\theta$$

where the spectrum function, $a(\theta)$, which I have introduced earlier, equals $\sqrt{S^2(\theta) + C^2(\theta)}$. Havelock further recalled

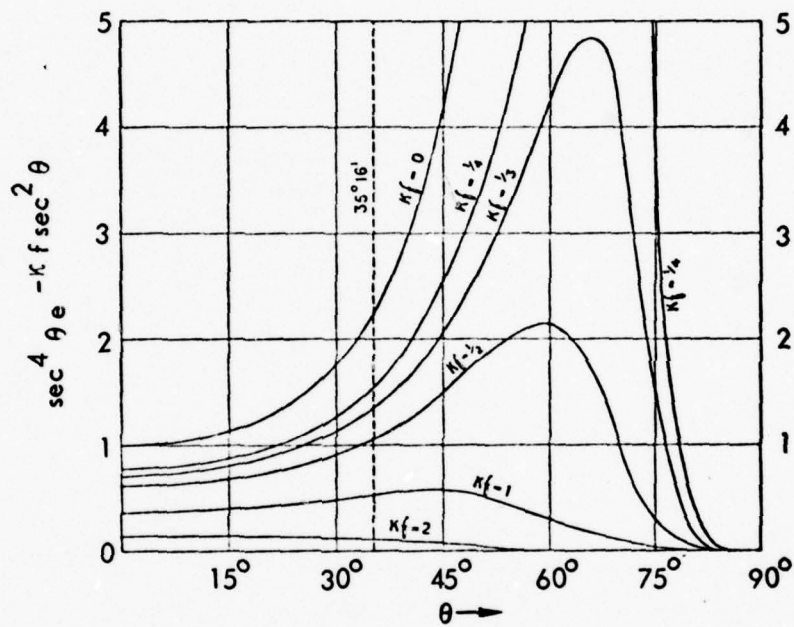


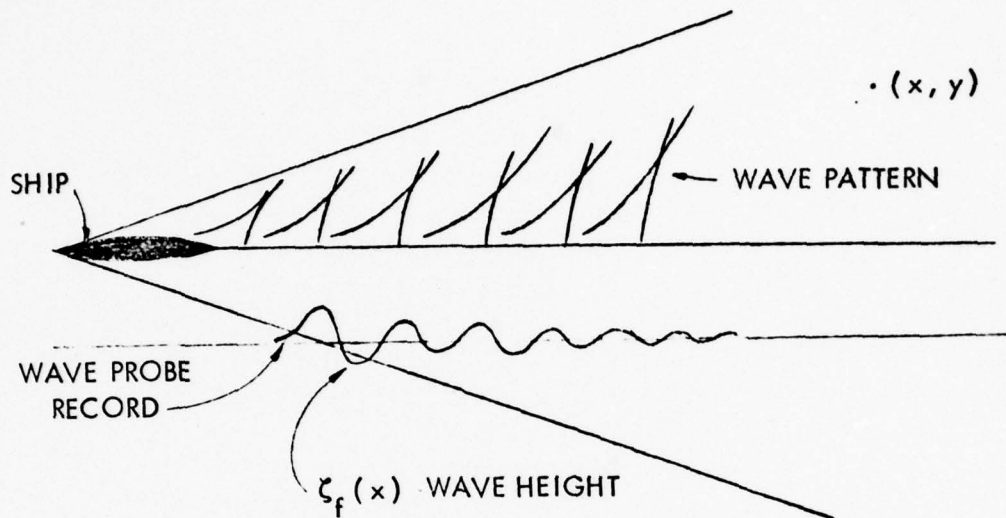
FIGURE 9 - $\sec^4 \theta e^{-k_f \sec^2 \theta}$ vs. θ , FOR k_f VARYING FROM: HAVELOCK (1934)

that the energy carried by the waves less the work done by them on the fluid ahead was equal to the product of ship resistance and speed, and he proceeded to calculate the net energy radiated in a general wave spectrum. He obtained the result:

$$\text{Resistance} = \frac{\pi}{4} \rho U^2 \int_0^{\pi/2} a^2(\theta) \cos^3(\theta) d\theta$$

showing how the contribution of the transverse waves (smaller θ) is heavily weighted.

Upon discovering this relation, Havelock commented, "It is rather curious that this method has not been used for obtaining the wave resistance from the wave pattern produced by ordinary ship forms." He referred, in fact, to the theoretical calculation of resistance, but the remark applies as well to its experimental determination through spectral measurements, a possibility which remained unrecognized until the mid 1950's when Korvin-Krokouvsy of the Stevens Institute urged it upon his colleagues. The task was first accomplished using wave probes in a towing tank by Ward (1964). See Eggers, Sharma, and Ward (1967). The method commonly used today involves a longitudinal sampling of the waves along a track transverse to the ship in its motion, at a sufficient distance abeam to avoid the effects of the local flow field, but close enough hopefully to obviate important effects of the tank walls. The oscillatory record so obtained is, in essence, Fourier analyzed to yield the spectrum $a(\theta)$, taking into account the finite sampling time, and if necessary the presence of the reflecting walls of the tank, Figure 10.



$$F_f(\lambda, y) = \int_{-\infty}^{+\infty} \zeta_f(x, y) e^{i\lambda x} dx \quad \lambda = \kappa \cos \theta$$

$$2\pi a^2(\theta) = \frac{2\kappa^2}{\pi} \left| F_f(\lambda, y) \right|^2 \sin \theta \tan \theta$$

WAVE ANALYSIS, TO OBTAIN AMPLITUDE SPECTRUM, $a(\theta)$
AFTER NEWMAN (1963)

FIGURE 10 - WAVE PROBE TECHNIQUE

The analysis theory often used is due to N. Newman (1963). The technique has been very successfully and ingeniously utilized by Japanese workers, especially of the Inui school, who well appreciate that a knowledge of the wave spectra offers invaluable information about the source of the waves and through a comparison with theoretical spectra offers powerful means of revealing the exact shortcomings of the theory, setting the way for its improvement. As mentioned earlier, I believe that the technique should be more widely used and could in time become part of a standard procedure in towing tanks for the dissection of ship resistance and its improvement through hull change. In doing so it must be remembered that in the near field of the ship, wave breaking and non-linear interactions (especially of the wave-current variety) can transform wave energy, thus affecting the far field spectra and the measurement of resistance; i.e., the probe measures only the wave resistance due to the radiation in the far field, which may not be the same in the real situation as the wave resistance on the hull. Experience shows that the latter is often larger.

Theoretical and wave probe spectra for a typical ship's bow followed by an infinite parallel body are shown in Figure 11, taken from Inui and Kajitani (1976). Note the relative transfer of energy from the lowest toward moderate values of θ for the real spectra in comparison with the theoretical, an effect called "sheltering", and the loss of energy at the high θ end. We shall comment further on these features later.

Two pressure points when separated in either the stream-wise or transverse direction will have amplitude spectra with

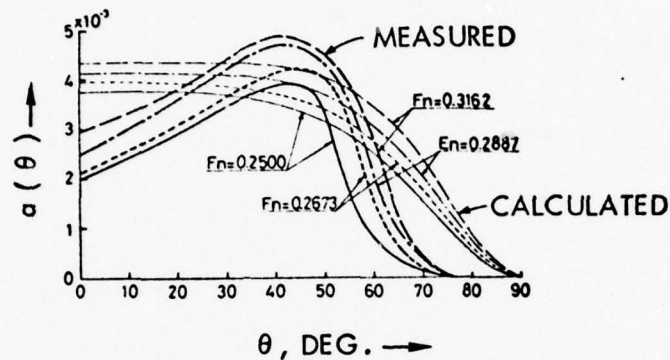


FIGURE 11 - COMPARISON OF MEASURED (ANALYZED) AND CALCULATED (LINEAR THEORY) AMPLITUDE SPECTRA, $a(\theta)$, FOR A SHIP'S BOW ALONE. FROM: INUI AND KAJITANI (1976)

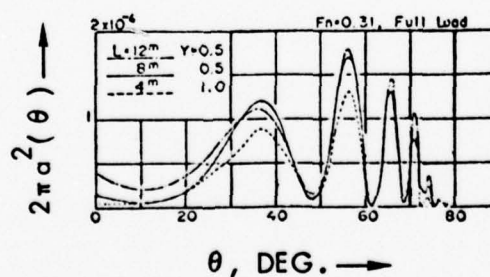


FIGURE 12 - AMPLITUDE SPECTRA FOR A TWIN SCREW CONTAINER SHIP. (L = MODEL LENGTH, Y = PROBE DIST. ABEAM) FROM: YOKOO AND TANAKA (1976)

oscillations, in contrast to the dipole spectra or the ship's bow spectrum shown above. These oscillations beginning in the diverging wave portion of the spectrum for small values of the non-dimensional spacing κs , spread to the transverse wave portion for sufficiently large spacings, Figure 12, where they are responsible for the oscillatory nature of the measured resistance curves of ships for $\kappa \cdot l > 4$ (i.e., $F_l < \frac{1}{2}$), and are therefore a manifestation of the interference between the bow and stern flows first elucidated by Froude.

The quantitative explanation of ship wave resistance curves in terms of the interference between synthetic planar surface pressure distributions representing the disturbances due to prominent features of the ship's hull, was pursued by Thomas Havelock in a long series of works between 1909 and 1921. Although explaining the important features of resistance curves, his approach, lacking account of the diverging waves and/or of the exact correspondence between surface pressures and hull shapes is finally semi-empirical.

The relative contribution to resistance from divergent and transverse waves may be calculated by dividing the integral over θ into two appropriate parts. Such calculations show that the divergent wave resistance increases at high Froude numbers and becomes dominant, see Figure 13.

The spectral superposition of individual waves and the use of their linear dispersion relations, as by Kelvin and Havelock, supposes in fact not only that the waves are small, but that so are the non-wave disturbances caused by the disturbing body or

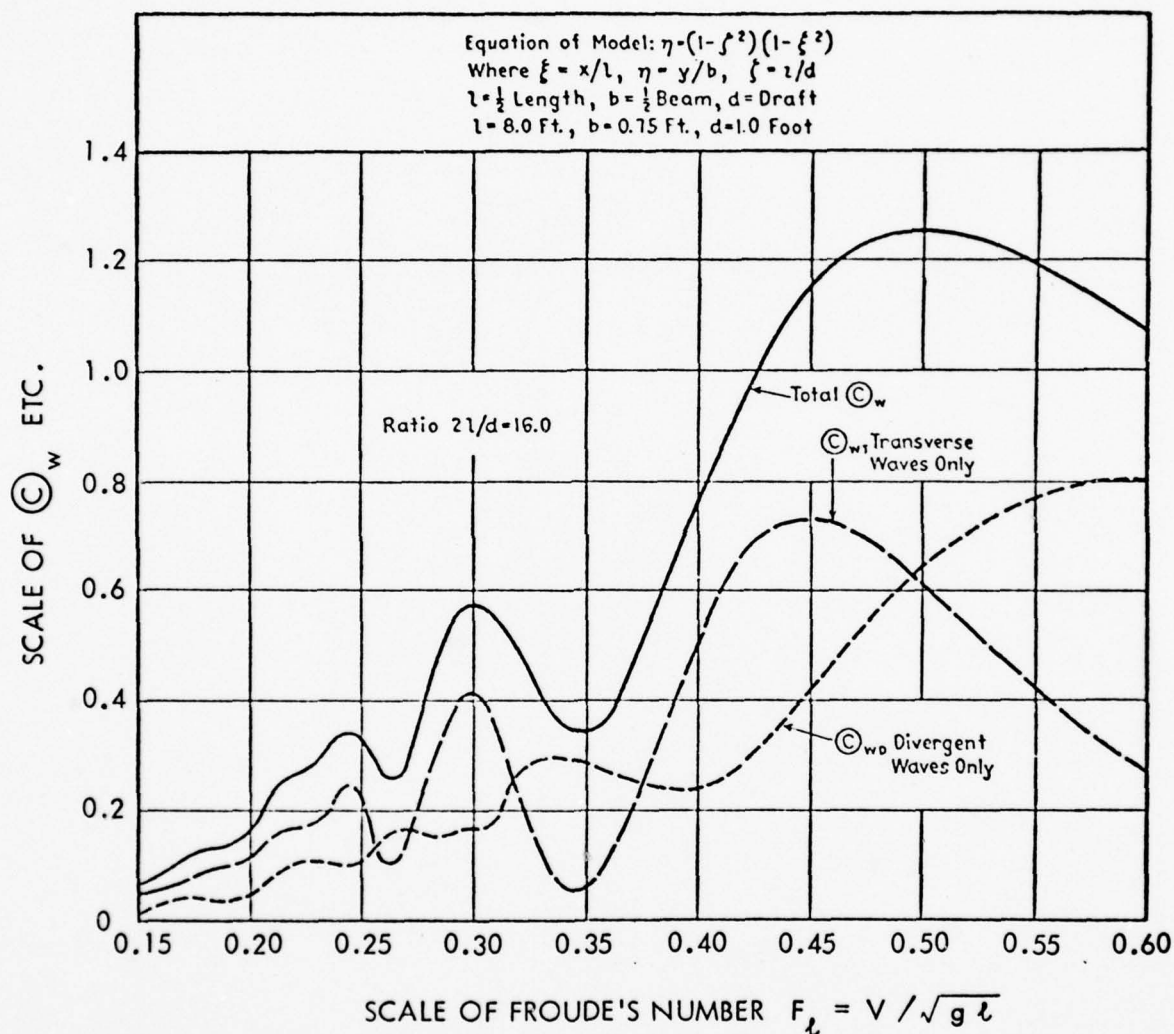


FIGURE 13 - CALCULATED WAVE RESISTANCE (MICHELL), SHOWING THE SEPARATE CONTRIBUTIONS FROM TRANSVERSE AND DIVERGENT WAVES. FROM: LUNDE (1951)

ship, hereinafter called the current field. Because of the decay of the wave amplitude with distance aft, and supposing that the current field decays, too, then the far field may certainly be represented by the Havelock spectrum, which has, too, all the kinematic characteristics of the Kelvin pattern. In the near field, however, non-linear effects (such as wave-hull, wave-current, and wave-wave, including breaking, interactions) may, in general, be expected, and must be evaluated. The calculation of the near field is the central remaining problem for theory.

THE NEAR FIELD: MICHELL, HAVELOCK, GUILLOTON, ET AL.

It is surprising how close to the ship a Kelvin-like pattern does establish itself, albeit with detectable distortion and wave breaking, see the Frontispiece. The caustics develop a slight concavity near the ship so that their effective origin is shifted forward about a half-beam. This distortion has been partially explained by Inui and Kajitani (1976) in terms of the diffraction of the wave pattern by the current field, á la Ursell (1960b). Further, the observed effect would seem at least related to that arising in the second order thin ship theory of Dagan (1975). In that theory, coordinate straining results in a forward shift of the first order hull singularities by a distance proportional to the beam.

The marked wave breaking invariably present around ships is probably often due to excessive wave slopes of the spectral components which are heavily weighted toward the diverging waves (which appear closer to the ship's track), and of the caustic waves. In the case of waves generated at sea by wind, Phillips (1958) showed that each spectral component has the same slope, limited by wave breaking. A ship wave spectrum of the same characteristic would have the form:

$$r^{-\frac{1}{2}} \cdot a(\theta) = (sl)^* \cdot \cos^2 \theta$$

where the limiting wave slope, $(sl)^*$, is $O(10^{-1})$. The rapid decay of this spectrum near $\theta = \pi/2$ is in contrast with linear theoretical predictions for ship forms, but is very suggestive of wave probe observations; these characteristically

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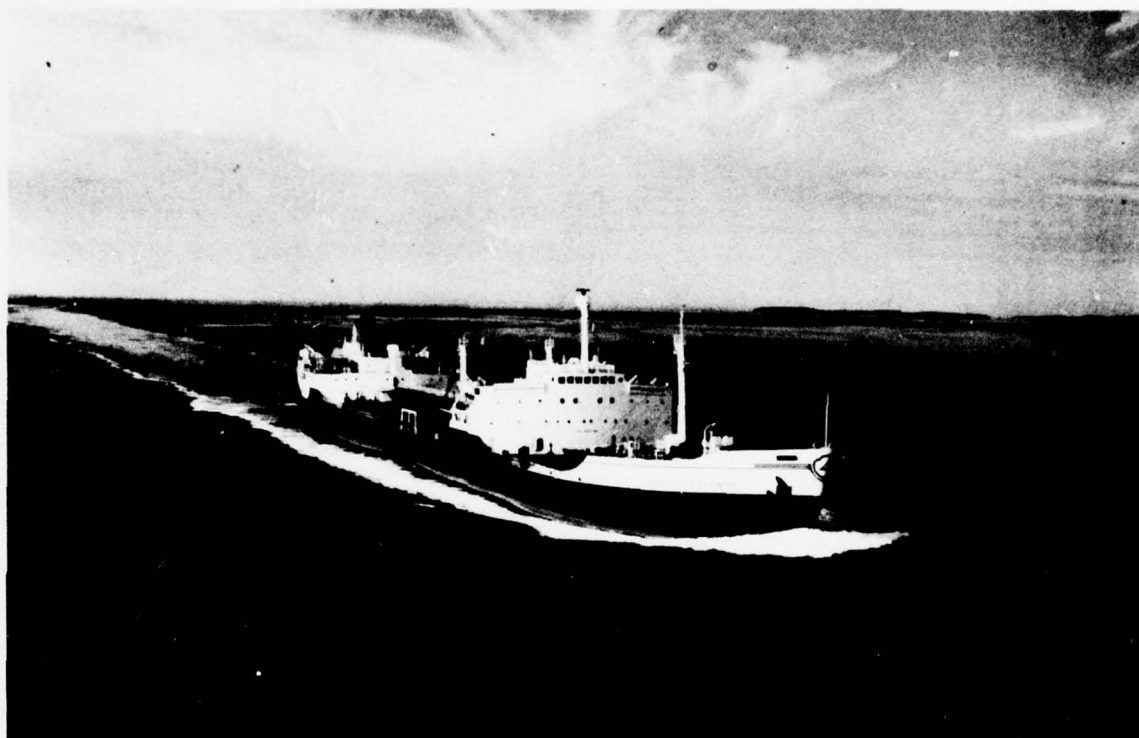


FIGURE 14 - A LOADED TANKER

show a vertical cut-off of the spectrum for angles greater than 75° , Figures 11 and 12. This cut-off effect is certainly in concurrence, too, with observations at sea which quite generally show a region of broken water centered about the ship's hull and track, as well as near the caustic. As the slopes decay aft like $r^{-\frac{1}{2}}$, these breaking regions are of finite extent. Near the caustics, breaking is often prolonged, as the wave amplitudes are enhanced and the decay, like $r^{-\frac{1}{3}}$, slower.

For ships of fuller form, as bulk carriers and barges, heavy breaking occurs forward of the bow, sometimes causing serious resistance, Figure 14. Its cause is still problematical. Bow wave breaking could conceivably be due to instability of the free surface, Dagan and Tulin (1969); to short wave steepening, Baba (1975); or to non-existence of the potential flow near the bow, Vanden Broeck and Tuck (1977). In the latter important work, devoted to the computation of the two-dimensional flow approaching or leaving a blunt box-like shape, it is concluded that "no continuous wave-free solution of the bow-flow problem exists;" instead a solution with a discontinuity in water height ahead of the bow is found.

Whatever the cause of wave breaking, and it is important to know, we shall not be able to calculate the breaking resistance of a tanker without at the least imbedding a model of wave breaking into the flow, akin to a re-entrant jet or spiral vortex, or something like that, as suggested by Tulin (1970), Figure 15, Dagan and Tulin (1970,72), and Vanden Broeck and Tuck (1977). The same remark applies even to slender ships or

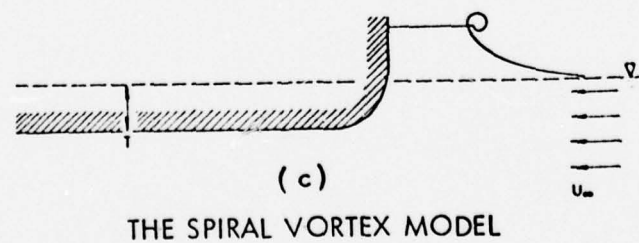
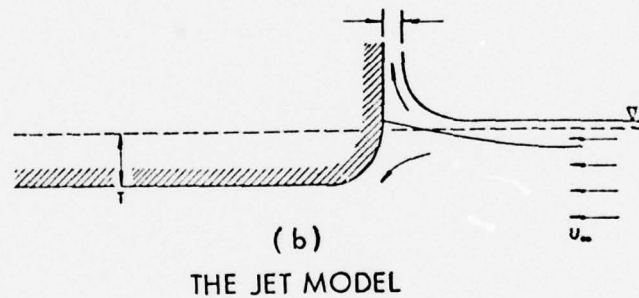
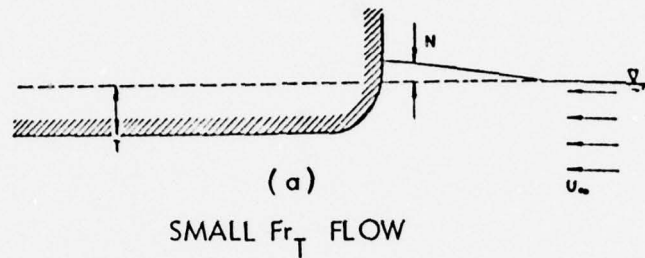


FIGURE 15 - THREE MODELS OF TWO-DIMENSIONAL FLOWS PAST BLUNT BODIES. FROM: TULIN (1970)

planing craft at sufficiently high Froude number, when wave energy is lost in spray, Tulin (1957). Nevertheless, all of the theoretical developments described in the subsequent sections ignore real fluid and breaking wave effects.

At the stern, frictional phenomena and the action of the ships screw especially influence the flow, resulting in a frictional wake flowing aft with decreasing intensity but increasing width. Its interaction with the waves in the near field, neglected with general success in the Froude procedure, is not yet quantitatively understood. While not pre-judging the importance of wave-wake interaction, I would point out that the importance of the ship's screw in thinning the stern boundary layer, in alleviating the tendency toward separation, and in returning the wake to an almost momentless condition must certainly be taken into account in any future studies of the subject.

It is in this physical context, surrounded by broken water and a turbulent wake, that the ship's hull in its motion gives rise to the wave field. The hope for theoretical solution has in the past centered about neglect of real fluid effects, leading to a potential problem with non-linear mixed boundary conditions, expressing the constancy of pressure, on the unknown free surface, and the usual Neumann (normal gradient of the potential specified) condition on the ship's hull. It is a tribute to Optimism that so many theorists have attempted the solution while every observation of a ship underway reveals broken water, often in profusion, Figure 16. I offer it as a serious problem to mathematicians with a taste for existence proofs to

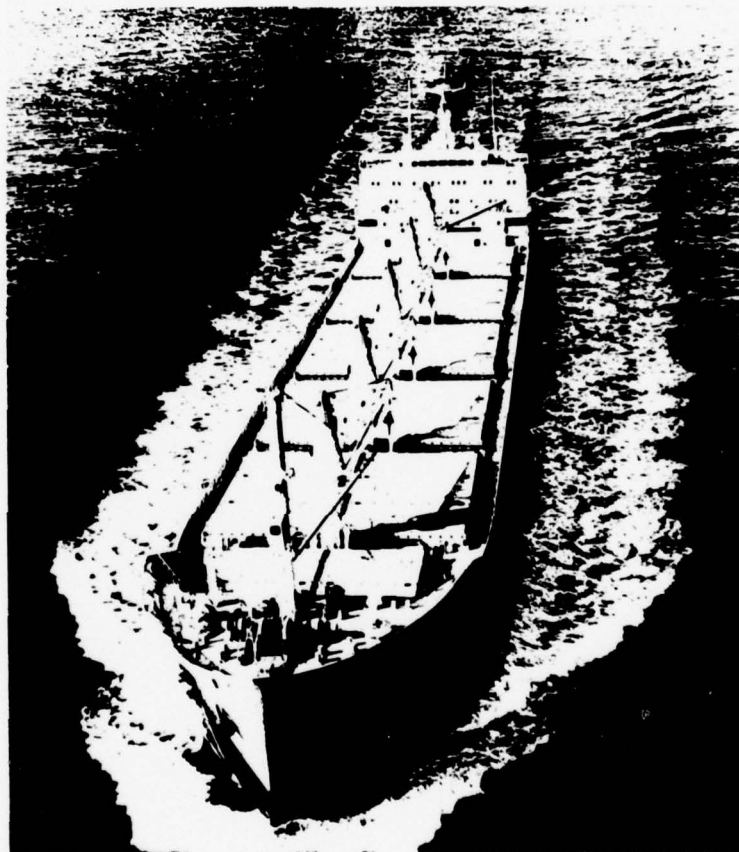
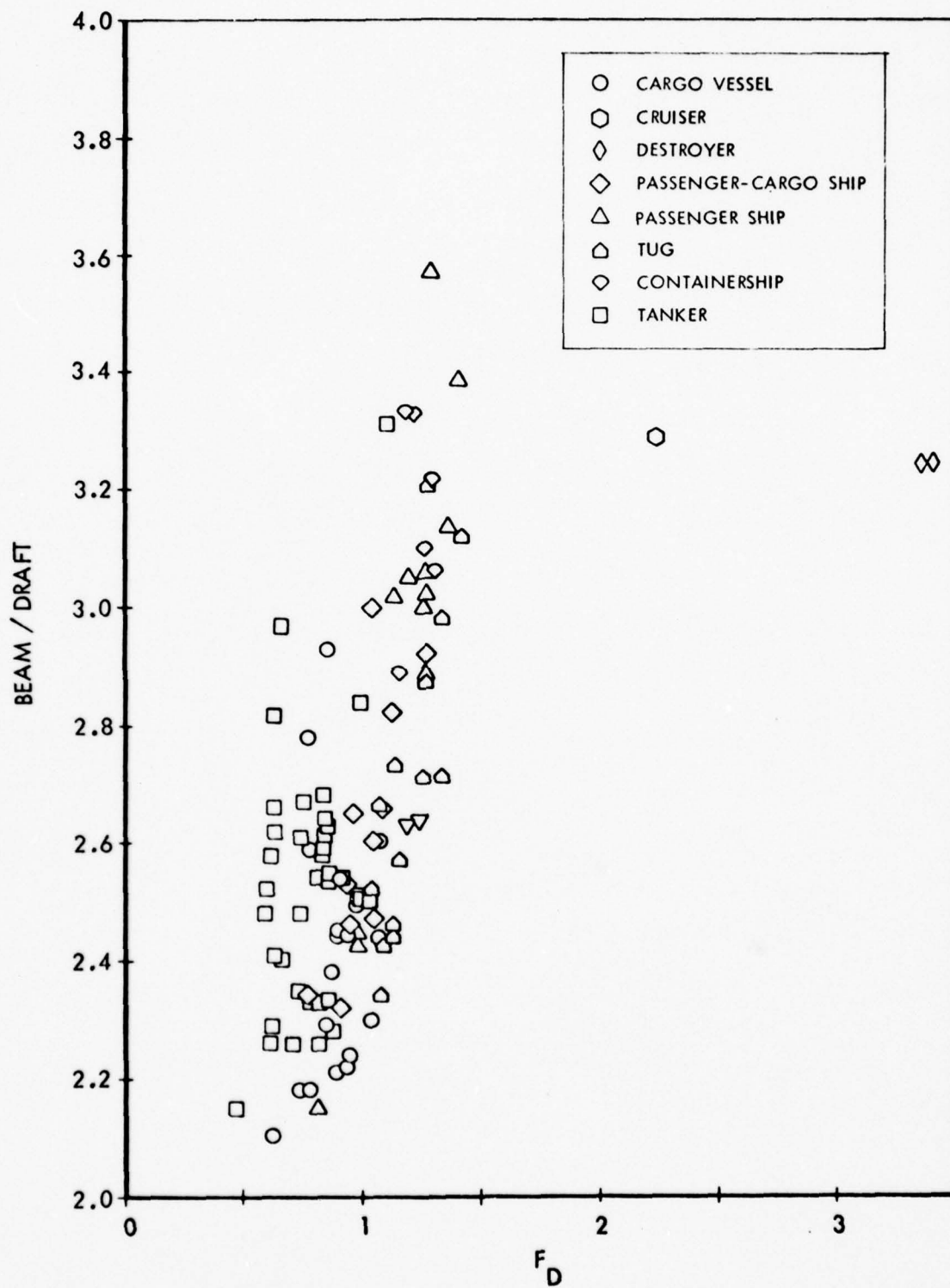


FIGURE 16 - HEAVY BREAKING

FIGURE 17 - B/D vs. F_D

demonstrate under what conditions, if at all, an exact solution to the usually stated problem exists for a ship-like body. Of course, no solutions are now known.

A typical ship in deep water may be characterized by its three principal dimensions: beam (B), draft (D), and length (l); while the operating speed is characterized by the length, λ , of the fundamental wave, or the wave number κ , where $\lambda = 2\pi\kappa^{-1}$ ($\kappa = g/U^2$). Typical ship dimensions are shown in Figure 17 and below:

	RANGE	"NORMAL"
$F_l = (\kappa l)^{-\frac{1}{2}}$	0.1 - 0.5	0.25
$F_D = (\kappa D)^{-\frac{1}{2}}$	0.5 - 3.0	1.0
B/D	2 - 4	2.5
l/B	5 - 10	7

(NORMAL: $\kappa l \sim 17$; $\kappa D \sim 1$; $\kappa B \sim 2.5$)

Do these dimensions strongly suggest any particular general approximation based on a small parameter expansion in one of the dimensions? Four major asymptotic theories have been proposed and/or developed to one degree or another:

NEAR FIELD ASYMPTOTIC THEORIES

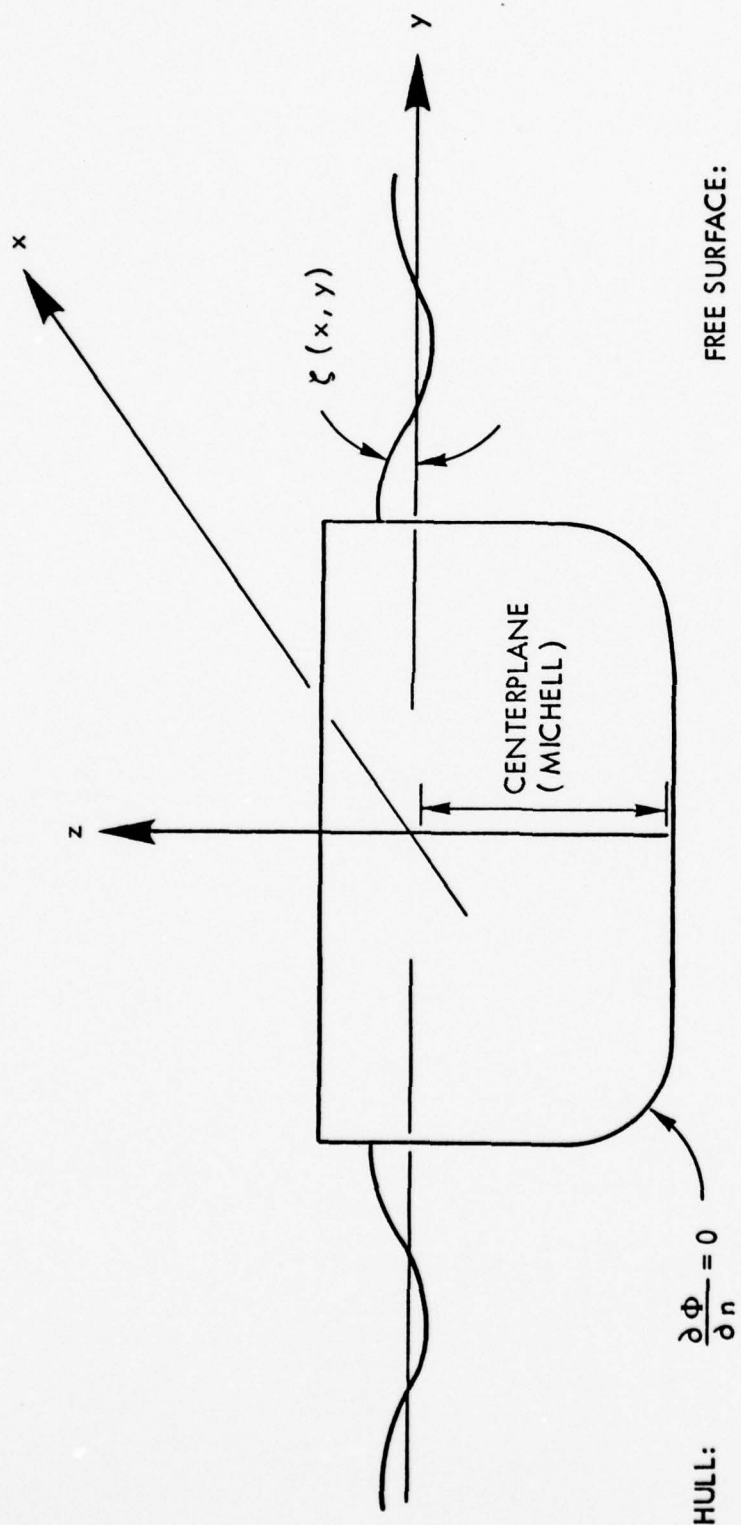
• THIN SHIP	$B \ll D, \ell, \kappa^{-1}$	DEVELOPED TO 2 nd ORDER
• SLENDER SHIP	$B \approx D \ll \ell, \kappa^{-1}$	DISCARDED IN FAVOR OF THIN SHIP
• FLAT SHIP	$D \ll B, \ell, \kappa^{-1}$	PARTIALLY DEVELOPED. INADEQUACIES
• SLOW SHIP	$\kappa^{-1} \ll B, D, \ell$	UNDER DEVELOPMENT

Of these the existing slender and flat ship theories suppose that the ship is placed immediately at the free surface, and therefore requiring $(\kappa D) \ll 1$; this requirement is especially important in view of the extreme sensitivity of wave spectra to the singularity depth, as we have seen, Figure 9 . And since, in normal practice, $(\kappa D) \approx 1$, the slender and flat approximations simply cannot succeed.

Thin ship theory was historically the first to approximate resistance curves of the general form observed. The theory including effects of finite water depth, was presented as a tour de force by the Australian, J. H. Michell, entitled "The Wave-Resistance of a Ship," published in the Philosophical Magazine in 1898. The boldness and ingenuity of Michell's solution is remarkable, and went twenty years beyond the state of art of the time. It refers specifically to a ship of small beam in comparison to other length scales. It thus represents the first

thin body theory in the history of fluid dynamics. It had an overwhelming effect on the field once it was recognized by Havelock in 1921, twenty-three years after its publication. The vast literature of application is well reviewed by Wehausen (1973). Its central result is an integral for the wave resistance in terms of the hull shape, which converges for shapes of practical interest. Fortunately, too, for usual wedge-shaped bows the theory predicts a finite wave height peaking just aft of the bow in the way generally observed, free of stagnation. The Michell theory, however, appropriate as it has been shown to be for a plank-like form of $B/l = (.05-.075)$ at sufficient speeds ($F_l > 0.2$), is according to experiments simply inadequate to predict accurately ship resistance of normal ships at usual Froude numbers, as carefully shown by Wehausen (1973). Nor have various ad-hoc attempts to render it useful via empirical corrects for viscous effects, sheltering effects, or others, been successful. Michell's theory predicts a resistance curve with more accentuated humps and hollows than observed and the predicted transverse waves at the stern of the ship and in its track are larger than observed; an effect called sheltering, which we have already observed in a comparison of experimental and theoretical amplitude spectra. Furthermore, the predicted bow wave peak is shifted aft of that observed.

In the thin ship approximation, the non-linear pressure condition on the exact free surface is satisfied instead on the horizontal undisturbed water surface in its linearized or Poisson form: $\phi_{xx} + \kappa\phi_y = 0$, while the Neuman conditions on the hull are satisfied as projected on the vertical center plane,



$$\Phi = Ux + \phi$$

$$V = |\nabla \Phi|$$

FIGURE 18 - SCHEMATIC OF BOUNDARY CONDITIONS

and then only in recognition of the streamwise slope of the transverse ordinate of the hull surface, Figure 18. As a result, the Michell boundary conditions are unaffected to first order by pitch of the ship's hull, contrary to experiments. Michell solved his problem by a method close to Fourier integrals. It is perhaps more physically descriptive to consider the solution as given by source-like singularities (i.e., Green's functions) distributed on the center plane with strengths proportional to the aforementioned hull slope, and independent of Froude number. The Green's functions satisfy the Poisson free surface condition and have wave-like properties in the far field, leading to resistance. The Michell solution may, alternatively, be thought of as the consequence of a superposition of the double-model flow ($F_\ell \equiv 0$) plus a continuous distribution of Kelvin sources over the whole horizontal waterplane whose strength equals exactly the linearized pressures caused there by the double model. This mathematical conception was foreshadowed by Wm. Froude's description of wave-making physics in 1875. He suggested first to consider the ship as traveling under a sheet of rigid ice (the double model) and went on: "If, now, we remove the ice, the fluid will obviously rise in level at each end, so that excess of hydrostatic head may afford the necessary reaction against the excess of pressure; and the fluid will sink by the sides, etc.... The hills and valleys thus formed in the water are, in a sense, waves; and, though originating in the streamline forces of the body, yet when originated, they come under the dominion of the ordinary laws of wave motion and, to a large extent, behave as independent waves."

The double model conception was not utilized or discussed by Michell himself but first by Horace Lamb (1913) who proposed an approximate solution to the two-dimensional flow past a submerged circular cylinder, in which the latter is replaced by a horizontal dipole of the same strength as pertains in the case of unbounded flow, plus its positive image reflected above the free surface, and superimposed on the free surface a distribution of pressure sources sufficient to satisfy the Poisson condition there, thereby cancelling the pressure due to the dipole. This solution is asymptotically correct in the limit when the depth of submergence becomes unbounded while the cylinder radius and wave number, k , remain fixed. Subsequently the same type of solution, obtained in analogous fashion, was presented by Havelock for submerged spheroids and ellipsoids. In the former case he showed his result to be closely similar, for slender bodies, to that obtained by application of Michell's theory, thus foreshadowing a result to be re-discovered much later in connection with slender body theory, Havelock (1923).

In the case of a circular cylinder, Havelock went on to add to the submerged dipole and its positive image, the reflections in the cylinder of the free surface image system which he had first described (equivalent in effect to the surface pressure singularities but located above the free surface at a distance equal to the submergence). This solution leaves the cylinder a streamline even while approaching the free surface. The resulting flow however, while satisfying the body boundary conditions exactly, do not satisfy those on the free surface beyond first order, and comprise therefore an inconsistent approximation, Havelock (1927).

Havelock (1928) presented for the first time, 30 years after Michell, the explicit form of the source Green's function satisfying the Poisson condition. In a later work (1932) he referred, albeit without enthusiasm, to two specific possibilities for going beyond the thin ship approximation of the hull boundary conditions, while retaining the Poisson condition. These were: a) to compose the solution of Havelock singularities, but with strengths appropriate to zero Froude number, and b) actually to determine the Havelock singularity distribution on the hull of the ship by satisfying the Neumann hull condition exactly, but the free surface condition only in its Poisson form. It remained for the Russian Kochin (1936) to present the appropriate integral equation in the latter case. This theory, once forgotten, has been re-invented in the last ten years, Brard (1972) for example, and for some reason tagged the Neumann-Kelvin (or NK) problem (Havelock-Kochin, or Neumann-Poisson would seem more logical). By whatever name, these problems posed by Havelock lead to inconsistent solutions, i.e., they are neither of first nor second order.

Do solutions based on the Poisson condition have any chance of success for real ships? I believe the chance is very small, since a variety of specific second-order calculations, Tuck (1965), Salvesen (1969), and Dagan (1973a), show that non-linear effects on the free surface, particularly phase shifts in the waves due to wave-current interaction, are just as and sometimes more important than non-linear effects on the body. These phase shift effects become more serious as the wave number increases (slower speeds), the body being fixed, and most ships do operate

at relatively low speeds. This situation has been analyzed in a series of papers by the Israeli Gideon Dagan starting about 1971. In particular, he concluded, Dagan (1972b), that the flow following from the Poisson free surface condition represents the first term in an expansion which is divergent for sufficiently low speeds, for fixed hull slenderness. The final conclusion regarding the range of validity of the NK theory awaits a serious comparison of calculated results and towing tests. Such a test has already been made of the double-model theory [Havelock's suggestion a) above] by Inui and Kajitani (1976). They have shown that the calculated resistance of ship hulls based on streamline tracing of artful Rankine singularity distributions (called Pienoids after P. Pien of the David Taylor Model Basin) do not adequately agree with measurements.

What then is the status of the consistent second order expansion of the thin ship? The Michell theory was formally recognized as the first order term in a small parameter expansion in B/l by Peters and Stoker (1954), Stoker (1957), who stressed that in going beyond Michell "it would be necessary to deal with the full non-linear problem, and make sure that all of the essential correction terms of a given order were obtained." The statement of the problem as a regular expansion was subsequently given by Sizov (1961) incorrectly, Wehausen (1963), Eggers (1966), and Maruo (1966), all in different form. Their connection has been shown by Kitazawa and Takagi (1976). All of these statements include surface integrals with kernels representing dipoles as well as sources, plus line integrals about the ship's waterline and, in the case of a flat bottomed

ship, the bottom-side intersection. However, an equivalent form in terms of surface integrals only, given by Dagan and Noblesse (1975), calls for a distribution of Havelock sources (only) on the undisturbed free surface as well as on the vertical centerplane, their strengths depending on the solution of the first order problem. One of the important second order effects seems to arise from the waviness of the free surface interaction with the wetted hull. Utilizing the line integral representation of this phenomenon, the Japanese Bessho (1976) has shown analytically that it corresponds exactly to the effect which would result should the hull be amputated immediately beneath the undisturbed waterline, and the flow be required to be horizontal along the then-flooded deck. This amazing result would seem to cause an inhibition of vertical motion near the ship's waterline in comparison to the first order prediction, a sheltering effect akin in nature to that observed, and would seem to provide a partial success for this second order theory.

The regular expansion theory as developed by Sizov, Wehausen, et al., has several weaknesses. Its solution converges non-uniformly in three aspects: 1) at singularities like the bow, stern and sharp shoulders where important waves originate and interact; 2) in the low speed limit ($\kappa \rightarrow \infty$); and, there is reason to believe, 3) in the amplitude spectra as $\theta \rightarrow \pi/2$, Dagan (1976).

It is well known that non-uniform solutions may be rendered uniform through co-ordinate transformations. Such transformations should obviate the necessity to expand the solution away from the surface on which the boundary conditions are expressed

and to render that surface planar. In two-dimensional hydrodynamic problems such a technique utilizes the complex potential (ϕ, ψ) rather than the physical (x, y) space, as in higher order cavity flow theory, Tulin (1963). In three dimensions, however, the field equation is not generally invariant even under orthogonal transformations, as noted by Yim (1968), who attempted the first such application to ship problems. His work was followed by Wehausen (1969) and Noblesse (1975) and Dagan (1975). These techniques lead to systematic procedures for determining the distribution of sources along the centerplane of the ship which generate a flow satisfying both the free-surface and body boundary conditions at second order. They nevertheless represent only first order solutions of the field equation, and are therefore inconsistent. Since the computations required to generate these inconsistent flows are, in fact, essentially the same as involved with Michell's theory, the technique has been to a certain extent evaluated experimentally. But therein lies a story.

In 1939, R. Guilloton presented a doctoral thesis at the Sorbonne proposing an application of Michell's theory involving two alterations: i) he composed the ship of a finite distribution of wedges with finite draft and infinite length, and ii) he identified the location of the wedge by a transformation based on the first order solution. The first of these alterations effects not at all the thin ship nature of the solution, but was designed to ease computations. In that pre-computer age, it was this alteration to which most attention was drawn. M. Guilloton, who subsequently pursued his study of ship waves

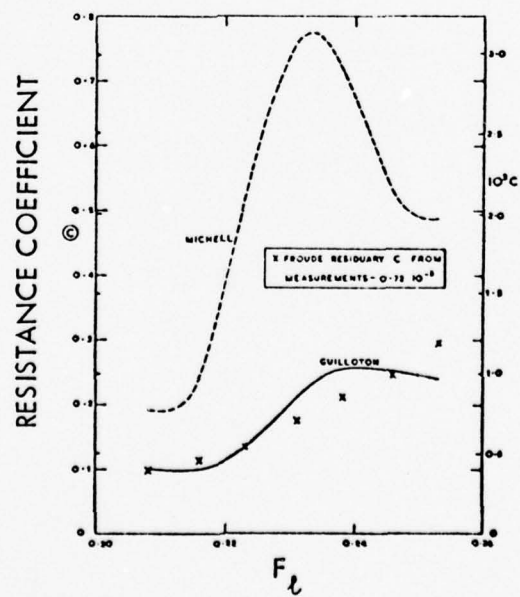
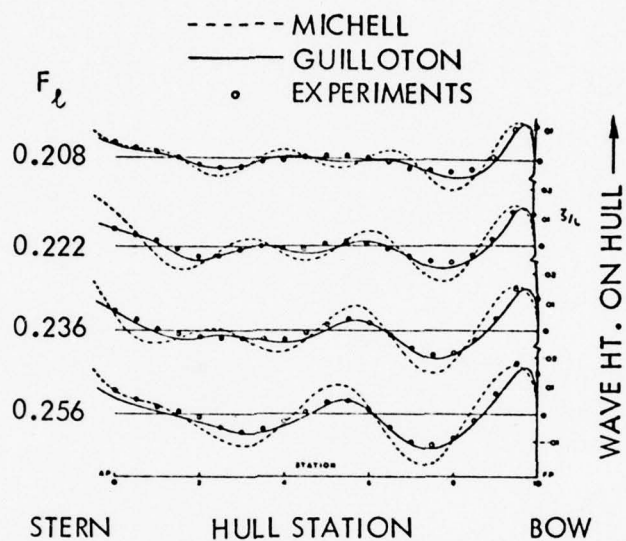


FIGURE 19 - WAVE PROFILES AND RESISTANCE
FROM: GADD (1973)

solely as an avocation, described his ideas in a series of papers in the period 1939-64, and in the last of these he suggested that his transformation technique resulted in boundary conditions being satisfied to second order on both body and free surface. This was proved by both Noblesse (1975) and Dagan (1975), whose own separate transformations are different but closely similar to that of Guilloton. The latter has been tested by Emerson (1967), Gadd (1973), and Standing (1974). The results of Emerson and Gadd are certainly very encouraging for ships of moderate prismatic, see Gadd's result, Figure 19, but in the simple case of fine wedge bows, (5° and 10° half angles), tested by Standing, the Guilloton method resulted in a worsened prediction (too low) of the bow wave amplitude than given by linear theory (too high); it did cause a forward shift of the wave pattern, but of insufficient magnitude.

This latter circumstance focuses attention on Dagan (1973a, 1975), who borrowed on Lighthill and van Dyke, for unlike all the other methods mentioned, where straining only begins from the bow, he strained the flow from upstream infinity. This results in a forward virtual displacement of the bow singularities of $O[B/l \cdot \ln(\kappa l)]$ for a wedge bow, therefore increasing with κ , as suggested in the discrepancy between Standing's data and Guilloton. In addition, this straining renders the solution uniform at the physical bow, and at the same time imbeds the speed dependent phase shift of the waves inside the wave form itself, rather than in the form of a supplementary wave appearing as an expansion term. This technique is illustrated schematically in the case of the resistance curve, Figure 20. Dagan

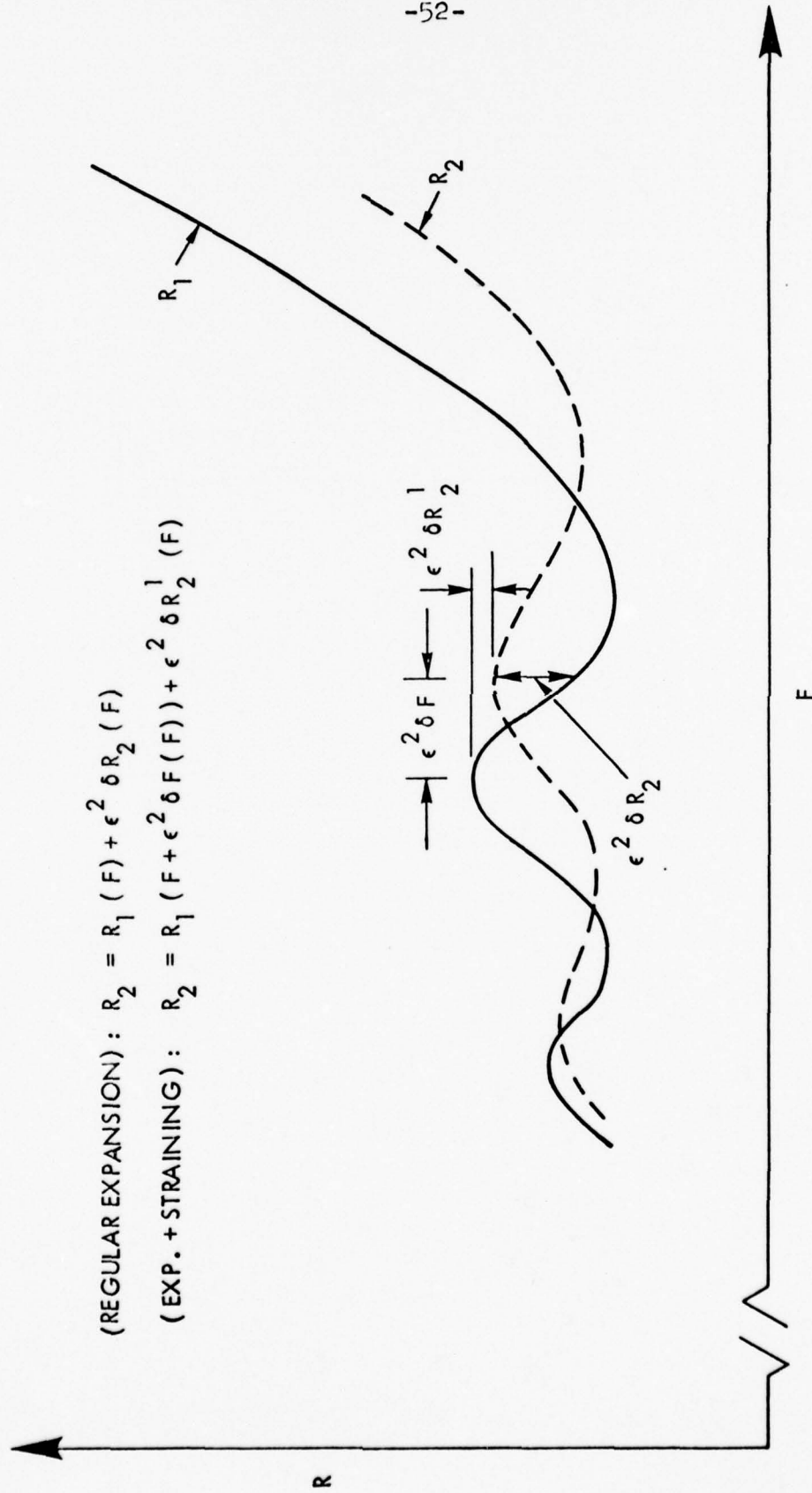






FIGURE 20 - ILLUSTRATION OF REGULAR EXPANSION vs. EXPANSION PLUS CO-ORDINATE STRAINING

claims that his technique can render the solution uniform with decreasing speed, unlike the regular expansion solutions. In the latter case, for ships with wedge shaped bows he found for the free wave potential:

$$\phi_1 \sim O[(B/l) \cdot (\kappa l)^{-1}]$$

$$\phi_2 \sim O[(B/l)^2 \cdot (\kappa l)^{-1} \cdot \ln(\kappa l)^{-\frac{1}{2}}]$$

so that the solution diverges as the speed vanishes, the ship's geometry fixed. Fortunately, the wedge bow non-uniformity is weak. Dagan emphasized, however, that the behavior of the second order expansion, and therefore the nature of the non-uniformity, depends on the bow shape. He did not present results for other bow shapes in the ship case (most ships do have wedge shaped bows), but he did analyze a number of two-dimensional problems. Here the slender body is completely submerged. The results, as shown in the Table below, are worth pondering:

TWO-DIMENSIONAL SUBMERGED		ϕ_2/ϕ_1 (FREE WAVES)	$O(\text{STRAINING})$
• SOURCE BOW		(BK)	(B/l)
• PARABOLIC BOW		$(\frac{B}{l}) (\kappa l)^{\frac{1}{2}}$	$(\frac{B}{l}) (\kappa l)^{-\frac{1}{2}}$
• WEDGE BOW		$(\frac{B}{l}) \ln(\kappa l)^{-\frac{1}{2}}$	$(\frac{B}{l}) (\kappa l)^{-1} \ln(\kappa l)^{-\frac{1}{2}}$
THREE DIMENSIONAL SHIP			
• WEDGE BOW		$(\frac{B}{l}) \ln(\kappa l)^{-\frac{1}{2}}$	$(\frac{B}{l}) \ln(\kappa l)^{\frac{1}{2}}$

What hope, finally, is there then that second order calculations whether based on regular expansions, Dagan straining, Guilloton, or other schemes proposed by Noblesse and Dagan (and not discussed here), to mitigate the inconsistent nature of Guilloton, will eventually work for normal ships? Answers:

- o The asymptotic study of Dagan teaches that normal Froude numbers and for wedge ends, the second order theory may possibly be applicable.
- o The chance of success at lower speeds especially, will be enhanced by Dagan straining (i.e., Froude number shifting).
- o For parabolic ends, or worse, the chance of success is doubtful, and the more so as these shapes are more likely to be used at lower speeds and to involve serious wave breaking.

Clearly, a very thoughtful and systematic evaluation, varying (B/λ) and (B/l) , embodying the best of second order theory, careful computations, and the best of experimental techniques, both near and far field wave measurements would be required at this time, in order to answer this important question.

"ESSO ATLANTIC"
AT - A - GLANCE

Length, OA, m	406.6
Beam, molded, m	71.0
Depth, molded, m	31.2
Draft, full load, m	25.0
Dead weight tonnage	508,731
Gross tonnage	234,627
Shaft horsepower	45,000
Service speed, knots	16



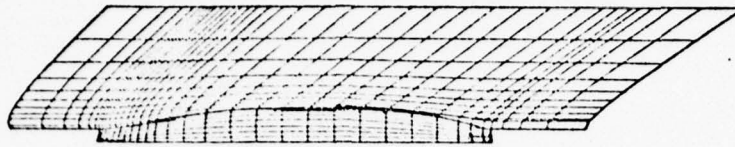
FIGURE 21 - A SLOW, FULL-BODIED SHIP

SLOW SHIP THEORY

Well, if the thin ship theory is hampered by low speed effects, why not try slow ship theory, whatever that is? A slow ship is shown as Figure 21.

It is, first of all, possible to pose a regular expansion in Froude number, sometimes termed the "naive" F expansion as it does not lead to waves, as shown by Ogilvie (1968) in the two-dimensional case. As if that were not trouble enough, it has recently been shown that the two-dimensional naive F expansion has a zero radius of convergence, Vanden Broeck, Schwartz, and Tuck (1978). The same authors showed that converged solutions obtained through the Shanks transformation possess jump discontinuities on the free surface forward of the blunt bow; thereupon they have averted to iterative solutions of the original boundary value problem, utilizing a portion of the converged expansion solution as known, and obtained smooth flows, but with waves at infinity. One importance of their work is that it confirms earlier doubt concerning the existence of smooth bow-like flows without waves forward at infinity. However, the generalization of their methods to the three dimensional case has not been approached.

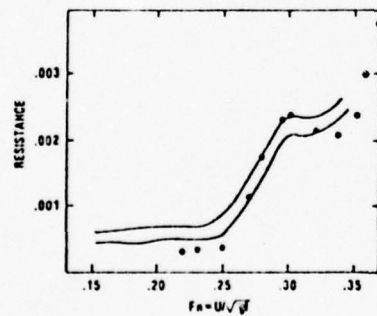
Much earlier, Ogilvie (1968) proposed to retain a zero order (i.e., double model) term in the free surface boundary condition, so as to produce waves; he treated a submerged planar body. In the resulting theory, the double model flow plays the role of the uniform flow in regular linear theory.



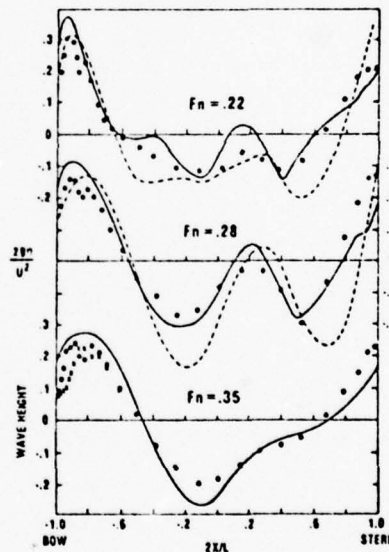
PANEL ARRANGEMENT FOR SERIES 60 SHIP

- PRESENT METHOD (WAVE RESISTANCE)
- EXPERIMENT⁹ (RESIDUAL RESISTANCE)

$$C_R = \frac{D}{\rho U^2 / 2}$$

WAVE RESISTANCE FOR
SERIES 60 BLOCK 60 SHIP

- HUANG & vanKERCZEK
- EXPERIMENT
- - - COMPUTED BY GUILLOTIN METHOD
- PRESENT METHOD
- PANELS NEXT TO BODY
- SECOND ROW OF PANELS

WAVE PROFILES FOR
THE SERIES 60 SHIPFIGURE 22 - DAWSON'S 'SLOW SHIP' RESULTS
FROM: DAWSON (1977)

Dagan (1972b), treated the slow ship in an important work, and showed that the appropriate slow ship theory ($\kappa^{-1} \rightarrow 0$, ship fixed, and of arbitrary B/l) involves the zero (Ogilvie) plus first order "naive" expansion terms in an appropriate boundary condition. He also derived a simpler form without the first order term and appropriate for $(B\kappa) = O(1)$, as is usually the case. Dagan's result, but omitting the first order term, was essentially re-discovered by Baba and Tukekuma (1975), Newman (1976), Maruo (1977), and, in a particularly nice form, by Dawson (1977) of the Taylor Model Basin:

$$\left(\phi_1^2 \varphi_1 \right)_1 + g \varphi_z = 2 \phi_1^2 \phi_{11} \quad (1)$$

where

φ is the total potential,

ϕ is the double model potential, and

l is the surface streamline direction for the double model.

His version is given in a paper entitled "A Practical Computer Method for Solving Ship-Wave Problems," in which he arrives at a slow ship approximation as a straightforward double model linearization (without mentioning slow ships at all) and showing some remarkable correlations between numerical calculations and data, see Figure 22. Dawson's method is based on satisfying the hull and free surface conditions through a distribution of Rankine sources distributed over panels on the hull and in a very limited region of the horizontal free surface near it; Figure 22, the computation for a dozen Froude numbers cost about \$500 and takes about one half hour of computer time. There are two

NAIVE F EXPANSION: $\Phi = \Phi_0 + F^2 \Phi_1 + \dots$

GENERAL SLOW SHIP BOUNDARY CONDITION ($z = 0$):

$$F^2 a_{ij} \varphi_{,ij} + \varphi_z = F^2 b(x, y) \quad i, j = x, y, z$$

	a_{xx}	a_{yy}	a_{xy}	a_{xz}	a_{yz}	$b(x, y)$
POISSON	1	-	-	-	-	-
OGILVIE, (1968), 2d	*	-	-	-	-	*
DAGAN, (1972 b), 3d; $B/\lambda = 0(1)$	**	**	**	**	**	**
BABA; NEWMAN; MARUO; (1976); (THEO.)	*	*	*	-	-	*
ABOVE; (PRACTICAL)	1	-	-	-	-	*
DAGAN, (1972 b); $B/\lambda < 1$; ($B\kappa = 0(1)$)	*	-	-	-	-	*
DAWSON, (1977)	†	-	-	-	-	†
		$(\Phi_{0,1}^2 \varphi_1)_1 + g \varphi_z = 2 \Phi_{0,1}^2 \Phi_{0,11}$				

* - BASED ON Φ_0 ; † - BASED ON THIN SHIP Φ_0 ; ** - BASED ON $(\Phi_0 + F^2 \Phi_1)$

1 - STREAMLINE COORDINATE, DOUBLE MODEL

FIGURE 23 - A COMPARISON OF SLOW SHIP THEORIES

differences between Dagan's prescription and Dawson's method:

i) in the free surface boundary condition, the omission by Dawson of first order terms; ii) in the hull boundary condition, satisfying it exactly instead of using the double model source strength; it would seem that these differences are of second order in the beam (B/l), and it would be most interesting to carry out Dagan's prescription exactly and to compare the results with Dawson.

The approach of Baba and Maruo is, in essence, not only to approximate Dagan by omitting first order F terms in the boundary conditions, Figure 23, but further to approximate Dawson in their final step by replacing his equation (1), pg. 58, with:

$$U^2 \varphi_{xx} + g \varphi_z = 2 \Phi_1^2 \Phi_{11}$$

This approach eliminates the possibilities to account for phase changes due to the interaction of current and wave fields by incorporation of the phase change within the wave form itself, an effect of increasing importance for larger B/l . Their theory, similar as it is in its form to second order thin ship theory by virtue of the integral of Havelock sources over the free surface which arises in its solution, raises the question as to the existence of a consistent theory in the intersection of the slow ship and second order thin ship. Does the theory of Baba and Maruo lie somewhere close to that consistent theory and does this fact account for such success as they may experience? Baba and Hara (1977) by further approximating the surface integrals in their theory, arrive at simple formulas for wave resistance

involving only a contour integration around the hull-waterline. They show remarkable agreement with resistance data for a range of conventional ship forms. Maruo and Suzuki (1977), however, in applying identical formulae, arrive at different numerical results and much less optimistic conclusions. The matter awaits resolution.

Meanwhile, Dawson's experience shows clearly the computational possibility to solve the consistent slow ship problem of the type formulated by Dagan, involving an integral equation over the water surface, without further approximation. It would seem possible, in addition, to evaluate with it the occurrence of wave breaking around the ship, and even to include in an approximate way, the influence of wave breaking on the surface boundary condition and therefore on the flow.

Finally, for sufficiently slow speeds (probably lower than for normal ships), asymptotic methods based on short wave approximations may prove successful, as they have been in optics. Keller (1974) has provided the beginning, by describing the kinematical effect of the current field about a very slow ship on the propagation direction and wave lengths of short waves.

NUMERICAL HYDRODYNAMICS

The three dimensional achievements of computational methods grow steadily. Important successes are i) the exact calculation of the potential flow about three dimensional bodies in the absence of a free surface, Hess and Smith (1966; 1973), and ii) the calculation of the forces and motions accompanying potential flow about complex floating structures in a seaway of small amplitude. Both of these calculations have been made utilizing surface distributions of Green's functions. In addition, the latter have been made utilizing hybrid techniques including finite elements. The latter have the potential advantage that they can be used in the absence of potential flow.

Gadd (1976) has approached the calculation of flow around ship hulls utilizing distributions of Rankine sources on the hull and in a limited portion of the free surface near the hull; exact boundary conditions are specified except that the free surface source distribution is placed on the horizontal plane, and an appropriate correction made. The resulting calculation is reminiscent of that required in Dawson's method. Gadd's results are, so far, encouraging; they include the flow about a very bluff bow. This progress, taken together with the growing and varied numerical approach to the ship wave problem evidences by the Second International Conference on Numerical Ship Hydrodynamics (Berkeley), augers well for eventual success. Of course, a few stumbling blocks stand in the way, such as wave breaking and transom stern separation; but likely these will be soon overcome. There will remain for the longer range, the inclusion of wake and propeller effects.

Will, then, purely numerical methods realize the goal of theoretical wave resistance predictions first envisioned eighty years ago in the remarkable mathematical work of J. H. Michell? That would still leave for more analytical methods the task of explaining further and in greater depth the complex and varied phenomena involved in the flow past ship forms and, perhaps, giving intelligent direction to the continual search for optimum hull forms.

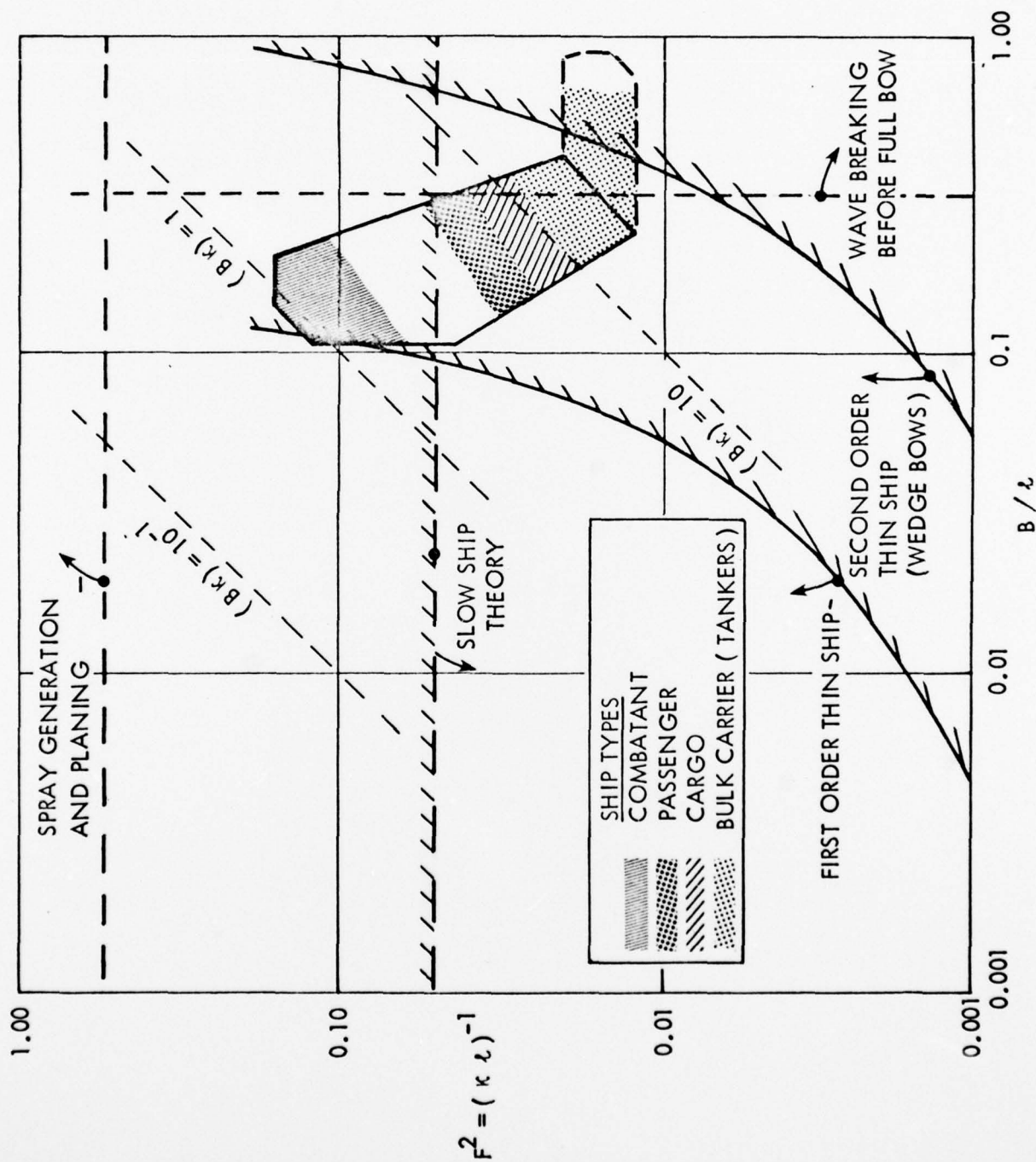


FIGURE 24 - THE SITUATION OF ASYMPTOTIC THEORIES

CONCLUSIONS

The current evolution of theory and its computer application, as well as of purely numerical methods, suggests that adequate methods for wave resistance and viscous wake estimation, for a range of ship types, could well exist within another decade, including the influence of breaking and the stern wake. The development of such methods would soon be followed by increased use of computer based estimations in hull design and powering.

The situation of theory, partly depicted in Figure 24, is:

- o Second order thin ship theory is both necessary and sufficient for slender high speed ships like destroyers and cruisers, preferably with Dagan-like coordinate straining. But two separate sources of trouble must be dealt with: i) the effect of strong divergent wave breaking along the bow and sides; ii) base effects arising from the transom stern.
- o For ships of moderate fullness and speed, as passenger and cargo ships, both inconsistent second order thin ship and moderate beam slow ship will sometimes give good results, separately, as in Dawson and in Guilloton. The proper theory, though, lies in the consistent intersection of

second order thin and moderate beam slow. A possible technique: linearization on a basic flow which is generated as second order in beam and second order in "naive" Froude number. In any event the calculation will then involve the numerical solution of a surface integral equation, as carried out by Dawson. The inclusion of wave breaking and wake effects will be possible.

- o Very full ships, like tankers, suffer wave breaking resistance, whose basic nature needs badly to be better understood. The full Dagan slow ship theory will allow the necessary calculations of the unbroken flow, of the wave resistance, and of the outer flow necessary to determine the wake, which is important for tankers. Adequate models of breaking, spiral vortex, re-entrant jet, or whatever must then be imbedded.
- o Purely numerical solutions of the exact problem, but allowing for wave breaking, do not now exist but could eventually deal with the general ship problem.

To put it all another way,

Naval ships are fast and slender,
Michell's close but needs a mender,
Second order fits the bill,
Dagan straining's better still.

Not too thin and not too slow,
Tells us how the freighters go.
Currents tag along the side,
Causing waves to shift and slide.
Poisson simply doesn't hack it,
Loses track of phase shift traffic.
Perturb rather on a flow,
Starting out as very slow.

Tankers cause a lot of breaking.
What's the cause this mess they're making?
Is it due to lost stability?
Or a sense of flow's futility?

Models now in tanks we tow,
All of that to Froude we owe.
Will computers, fast and new,
Make us alter Euler's view?

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